S2ShapeIndex Validation

Better living through invariants

Provide the ability to validate shapes in an S2ShapeIndex according to one of several correctness models.

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Goals

Build functionality to validate the geometry in S2ShapeIndex instances with a configurable model of correctness, with an ultimate goal of providing functionality to replace S2Polygon IsValid, and implement an ST_IsValid predicate.

Motivation

S2Polygon provides an IsValid() function that checks a number of invariants that are expected to hold. STGeography is valid *by construction*, and S2Shape doesn't have a validation function at all. We don't have the ability to take an arbitrary S2ShapeIndex and verify it meets the constraints to be treated as a particular geometric object. We'd like to be able to take an S2ShapeIndex containing arbitrary geometry (points, lines, and polygons) and validate it as a single unit with a configurable model of correctness.

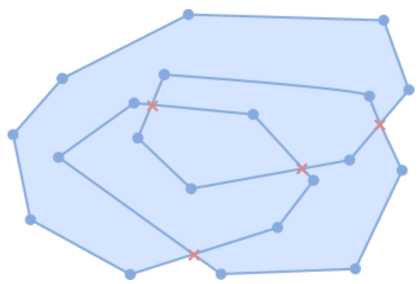
The goal isn't necessarily to validate the consistency of an index itself, but rather to validate the geometry *in* the index to verify that it can be used in situations where e.g. an S2Polygon or an STGeography are needed.

Background

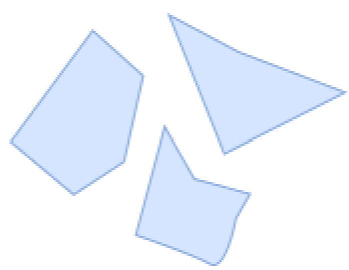
Why Validation

The purpose of validating geometry is to ensure that it meets some set of constraints that let us reason about it and make statements such as “point X is contained in polygon Y” robustly. It’s beneficial to think of geometry topologically, as sets of points that divide the domain they occupy. We can define multiple partitioning schemes (such as points in the interior, exterior, and boundary). But the simplest mental model is to view geometry as point sets partitioning space into two pieces: points that are part of the geometry and points that aren’t.

In the most lax sense, valid geometry is just geometry where points are unambiguously members of at most one of these two sets. This immediately forbids polygons with self intersecting edges:



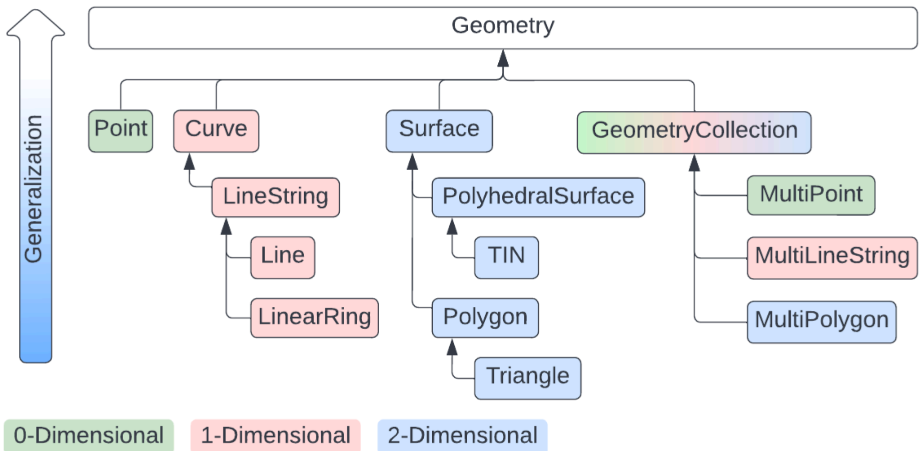
Not least because determining actual points of intersection is numerically difficult, but because we have to make up an arbitrary rule to determine whether the overlapping regions are part of the interior of the geometry or not. Note, however, that polygons with multiple disconnected interiors are allowed under this definition, since there’s no ambiguity:



For S2, it’s always possible to rebuild geometry with e.g. S2Builder to remove any degeneracies and ambiguities such as duplicate edges, reverse duplicate edges, or polygons with overlapping interiors, but this can often be cost prohibitive, especially if it’s known that the geometry was originally valid. We would like to be able to simply check that it remains valid according to a specific model of correctness and continue using it. Validation lets us verify our assumptions and operate on geometry with confidence.

OGC Data Model

The Open Geospatial Consortium (OGC) defines a standardized hierarchy of geometric types including points, line strings, polygons, and more specific subtypes[1]. It also defines a set of "Collection" types which aggregate geometry together. The type hierarchy can be seen in the image below, which shows the types and their dimensions.



A 0, 1, or 2-dimensional S2Shape would correspond to a MultiPoint, MultiLineString, or MultiPolygon, respectively, and an S2ShapeIndex containing many shapes could be any of the above, or a generic GeometryCollection containing a mix of dimensionality.

Note that the OGC types specifically require linear interpolation between points, which means that edges are "straight lines" in the chosen spatial reference system.

Using straight lines as edges has shortcomings when working with geographic data, which is referenced to the surface of an ellipsoid. To address this, the Geography[2] type was designed, specifically representing edges as geodesics.

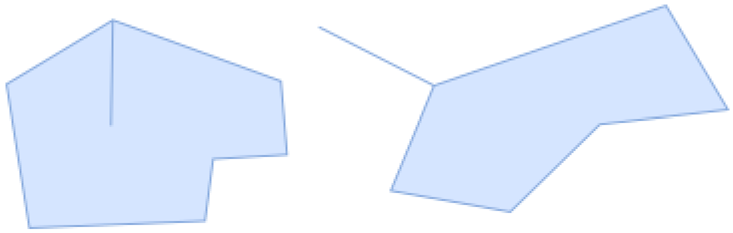
Validity

There are some geometric configurations that are simply not allowed by the OGC spec. Beyond the basics of requiring points and vertices to be non-infinite and non-nan, there are restrictions on some types we must check for ST_IsValid to be true. Points and LineStrings are inherently valid (though not necessarily simple as we'll see). Polygons, however, have several restrictions to make them topologically reasonable:

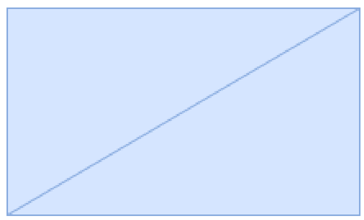
- May not have any cuts or spikes as they affect closed-ness.
- Chains must be closed.
- Chains may not cross and chains may touch but only at a vertex.
- Polygon interiors cannot be split (i.e. they are one connected point set)
- Chains can't contain each other (i.e. they must face each other properly).
  - Said another way the winding number of any point must be 0 or 1.

[1] https://www.ogc.org/standards/sfa
[2] See PostGIS Geography Data Type

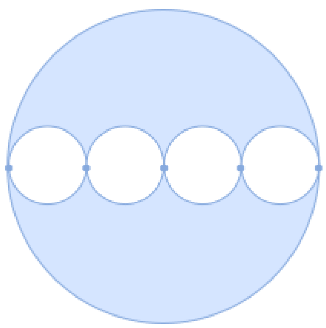
Cuts and spikes in a polygon are forbidden because they can make the geometry an open set (i.e. not equal to its own closure):



Split polygon interiors are also forbidden, and interiors can be divided in non-obvious ways that have to be detected, such as a single edge crossing the interior:



Or, being split by a chain of connected holes:



Simplicity

The OGC Simple Features specification[3] (SFS) differentiates between geometry that is merely *valid* and geometry that is *simple*. Simple geometry is always valid but the converse isn't true. A simple example of this is a MultiPoint containing the same point twice. This doesn't qualify as *simple*, but it is *valid*.

Simple geometry is constructed in such a way that it's possible to unambiguously categorize every point as being in the *interior*, on the *boundary*, or in the *exterior* of a feature. These three categories of points are used to define relationships between different features using the dimensionally extended nine intersection model (DE-9IM)[4].

For the built-in OGC types, the rules to ensure simplicity are summarized here:

[3] https://www.ogc.org/standards/sfa
[4] See §6.1.15.2 of OGC standard above.

- Point - Points are always *simple*.
- MultiPoint - MultiPoints are *simple* if no two points are equal.
- LineString - LineStrings are *simple* if they don't pass through the same point twice, excluding the endpoints for a closed LineString.
- MultiLineString - MultiLineStrings are *simple* if all their elements are simple and intersections between any two members occur on the boundary (endpoints, closed LineStrings may not touch)
- Polygon - A *valid* Polygon is *simple*.
- MultiPolygon - MultiPolygons are *simple* if all members are *simple*, no two members have intersecting interiors, and any two members that touch do so at a finite number of points (i.e. not along an edge).

Ultimately, we'll want to be able to determine whether geometry is *simple*, *valid*, both or neither.

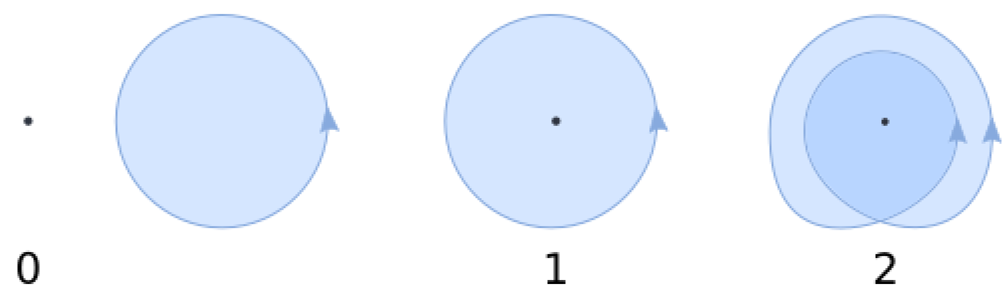
STLib

Validity

The OGC spec allows GeometryCollection subclasses to place additional constraints on the overlap between elements, but PostGIS elects not to for Geography, instead treating it simply as a regular GeometryCollection with an appropriate spatial reference system id (SRID) set[5]. Simple and valid checks are applied independently to each member element, and elements are allowed to overlap. This presents issues when we wish to process the geometry robustly.

Higher Kinded Regions

We can classify a particular pointset by the winding number of its points. Points that are in the exterior of all geometry have a winding number of 0, and those that are contained once by a polygon have a winding number of 1. Additional nested containment increments the winding number each time.



The DE-9IM models relationships between regions of winding number 0 and 1, and the boundary between them, for a total of three region types. It's possible to extend this model[6] to accommodate regions with higher winding, but the number of relationships to track grows extremely quickly.

[5] See PostGIS Source Code
[6] Categorizing Binary Topological Relationships Between Regions, Lines, and Points in Geographic Databases, Engenhofer

The DE-9IM has 9 total terms and thus 2^9 = 512 relationships. A model supporting winding 2 regions and additional boundaries would have a total of 6 types of region, for 36 total terms, or 2^{36} = 68,719,476,736 relationships. Modeling winding 3 regions as well puts the number of relationships over 10^{30}.

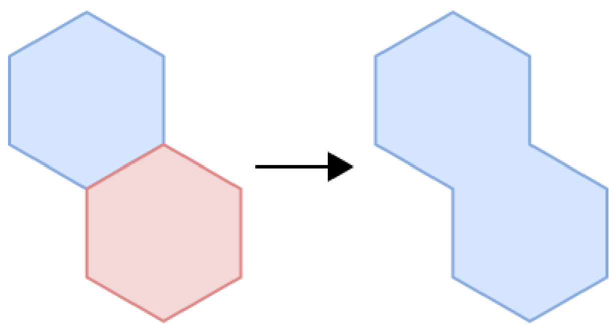
The vast majority of these relationships do not have a meaningful name in human language and so aren't terribly useful when trying to do "real work" with geometric data. Even the DE-9IM only names ~10 of its 512 relationships. In the interest of tractability, it behooves us to limit ourselves to regions of winding 0 and 1.

To avoid needing rules for higher-winding regions, STLib adopts a rule intended to disallow them altogether. Geometry in a particular Geography may not overlap in such a way as to have redundant information. Representing multiple overlapping layers requires separate Geography instances for each layer.

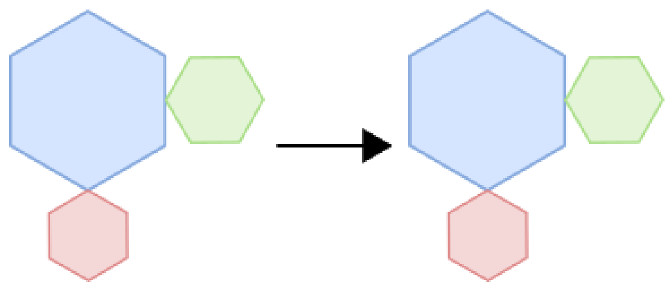
Dissolving

To enforce this, STLib operates on *dissolved* geometry, which is a canonicalization process that removes ambiguities and redundancy in the data. It's implemented as a cross-dimensional *union* operation that merges geometry and only leaves geometric features that add information by extending the covered point set in some way.

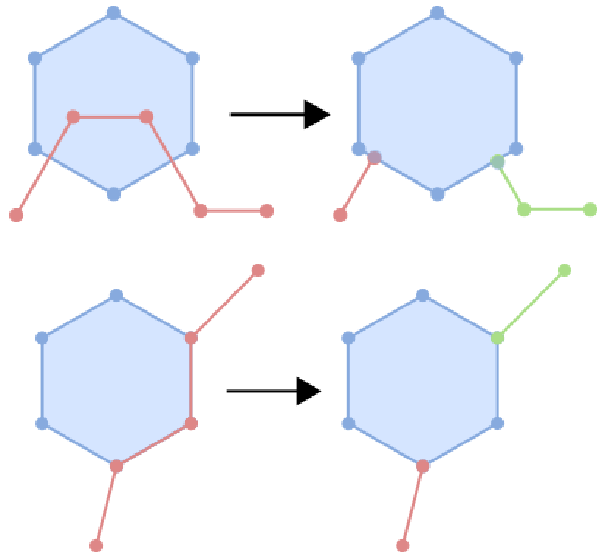
For example, when two polygons share an edge, the extra edge adds no additional information and raises questions about the closed-ness of the polygon, so it's removed:



Sharing a single point on the boundary, however, does not affect the closedness of the interior, so it's left unmodified:



When a higher dimensional object overlaps a lower dimension object, the overlapping region doesn't add any additional information, so it's removed, with the lower dimensional object being split as needed:



Simplicity

These rules are intended to prevent overlapping geometry, so that a given Geography instance partitions the world into two sets: contained and not contained. In general, this means that elements of a Geography *must* be *simple*, and we also disallow features that don't extend the covered point set in some way.

Additional constraints that STLib requires:

- Reverse duplicate edges aren't allowed either since polylines can't overlap the boundary of a polygon and polygons can only touch at a point already. Duplicate edges are always forbidden.
- Points may not overlap any other points or vertices.

Eventually, we will allow polylines that self intersect as in a GPS track, in which case we will have to have a separate check for self intersecting polylines to verify simplicity.

S2Polygon Validity

S2Polygon::IsValid() does the following checks:

- Vertices must be unit length.
- A loop must be empty, full, or have 3+ vertices.
- A loop can't have duplicate adjacent vertices.
- A loop can't have antipodal adjacent vertices.
- Loops can’t share edges (no duplicate or reverse-duplicate edges).
- A polygon can't have loops with zero vertices.
- A polygon can have the full loop iff it's the only loop.

To ensure the polygon is suitable for point containment queries:

- A Polygon can’t have crossing loops.
- And no loop can have crossing edges.

S2BooleanOperation Validity

As opposed to S2Polygon and OGC Geography, the requirements for geometry to be valid for use with S2BooleanOperation are comparatively lax:

Required

- Polygon interiors must be disjoint from all other geometry.
- Duplicate polygon edges aren’t allowed (even among separate polygons)

Allowed

- S2ShapeIndex *may* contain any number of points, polylines and polygons
- Point polylines composed of a degenerate edge AA
- Point loops composed of a single degenerate edge AA
- Sibling edge pairs such as {AB, BA}
  - These may represent shells or holes.
  - Or may represent separate polygons touching.
- Points and polyline edges are treated as a multiset and may have duplicates
- Polylines may have duplicate edges and self intersect.

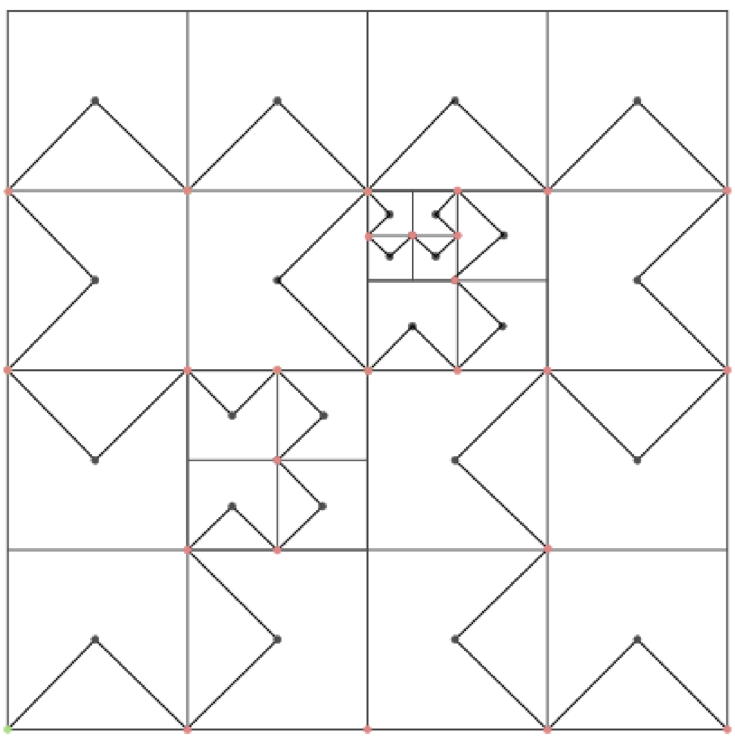
Interior Tracking and Edges

We'll discuss briefly how interior tracking in indexes work and why this imposes constraints on the type of edges we can have (i.e. duplicates and reverse duplicates).

Each shape in an index contains a flag indicating whether the center of a particular cell is contained in the shape. Put another way, we create a mapping of S2CellId => [ClippedShapes] when building an index, and each clipped shape knows whether the center point of the cell is in the interior or exterior of the shape. This enables queries such as S2ContainsPointQuery to draw a line between the query point and the cell center, and merely count edge crossings to determine whether the point is inside or outside of the shape.

Conceptually, tracking each shape's interior is done by drawing a continuous curve between cells as we build the index. We define an *entry* and *exit* vertex for every cell based on its *ij* coordinates and move from the entry point to the child cells, to the cell exit point. The entry and exit points are chosen so that the *exit* vertex of one cell is the *entry* point of the next cell in the hilbert order.

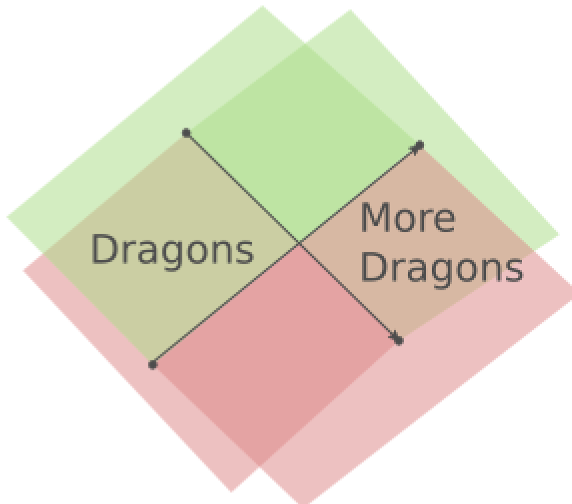
This ultimately gives us a curve that continuously moves between the cell centers:



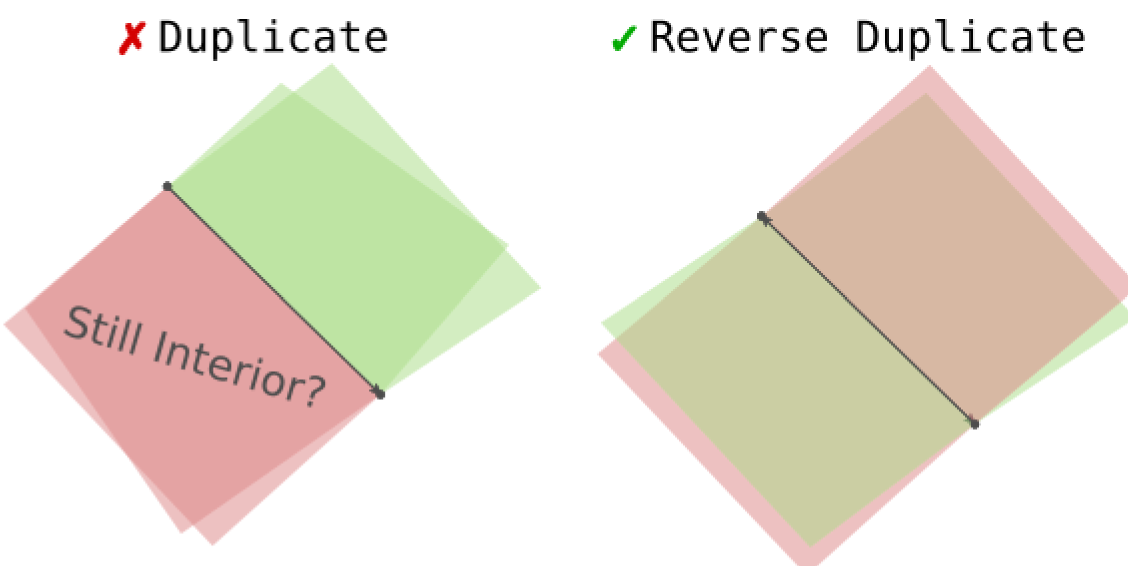
As we move the curve through a cell, we track which edges we cross and toggle the containment flag for the corresponding shape.

The net result is that no matter how a polygon is distributed amongst cells in an index, we always have an accurate accounting of its interior. *But*, this requires that every edge we encounter moves us from the interior to the exterior or vice versa.

This 1:1 requirement between edge crossings and interior transitions is what gives way to the rule that we may not have duplicate edges and that polygons must not self intersect. To visualize this second case, just imagine the interior/exterior arrangement of any two crossing edges from the same polygon. Interiors are on the left, so the opposite side of each edge must be the exterior. No matter how we arrange it, we can't make crossing edges consistent with the 1:1 rule:

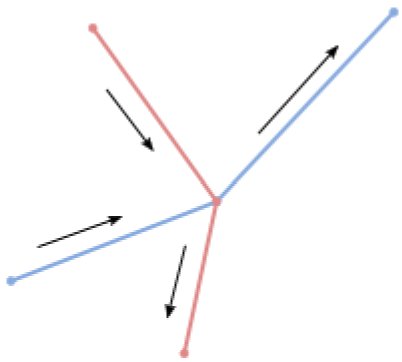


Similarly, duplicate edges require us to toggle twice, so we remain in the interior or exterior, but since interiors are on the left, we can't have interior or exterior on both sides of a double edge, because one side will always be oriented incorrectly. *Reverse* duplicate edges, however, present no such contradiction and are allowed.

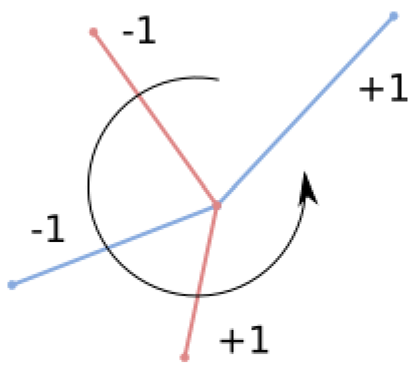


What's more, even between polygons, duplicate edges represent a double containment issue, because we have two interiors on the left, and polygon interiors must be disjoint, so duplicate edges are prohibited everywhere.

As far as actually detecting crossing edges, interior crossings are comparatively simple. We can check each edge pair in a cell and see if there's an interior crossing point with S2EdgeCrosser. But how do we detect whether there's a crossing at a vertex? Such as this situation:



The answer is that we can number the incoming edges as -1 and outgoing edges as +1, and visit them in a counter-clockwise order. We scan forward from an outgoing edge to its subsequent incoming edge and if any chains have summed to a value other than zero at that point, then we know we've got an edge intervening and thus it must be a crossing from one interior to another:



Note that, to support reverse-duplicate edges, we must be consistent in how we order edges with their siblings. We take the convention that outgoing edges in a sibling pair must come first in CCW order. This means that if we start at an outgoing edge, we'll always encounter its reverse duplicate on the next edge and stop scanning, with no other edges able to intervene.

Design

Predicates

Predicate	Description
ContinuousEdges	edge[i].v1 == edge[i+1].v0
MinimumChainLength	0D shape chains must have exactly one degenerate edge (a point), 1D and 2D chains must have at least one edge.
NoAntipodalEdges	No edges with antipodal vertices
NoChainCrossings	Polygon chains do not cross, they may meet at a point.
NoDegenerateChains	2D shape chains must have at least 3 edges
NoDegenerateEdges	1D and 2D shapes may not have degenerate edges
NoDuplicateChainVertices	Chains may not contain the same point twice, excluding the endpoints when closed.
NoDuplicateEdges	No duplicate edges at all.
NoDuplicatePoints	Any points contained in a 0D S2Shape must not overlap any other point or vertex and must be represented only once.
NoDuplicatePolygonEdges	No duplicate edges {AB, AB} between or within polygons.
NoReverseDuplicateEdges	No reverse duplicate edges at all (e.g. {AB, BA})
NoSelfIntersection	Polylines may not self-intersect, excluding endpoints. MultiPolylines may only intersect at endpoints.
NoSplitInteriors	The interiors of polygons are topologically connected (ie no pinch points)
NothingContained	No geometry is contained in the interior of any other.
OnlyPolylineEdgesCross	Only polyline edges may cross, and only to self-intersect.
PolygonClosed	If a 2D shape chain has more than one edge, it must have at least 3 edges and be closed.
PolygonFullOk	A polygon may have an empty chain if it's the only chain
PolygonsTouchAtPoint	Polygons may not share edges, duplicate or reverse duplicate. They may touch at a point.
Single2DShape	Only one shape is in the index, and it is 2D
ValidCoordinates	Points/Vertices aren't inf or NaN and have unit magnitude
ValidDimensions	S2Shapes have valid dimension value

We can distill these requirements into a list of predicates that we need to show are true for a given set of geometry. Different subsets of predicates will be used for different validation semantics.

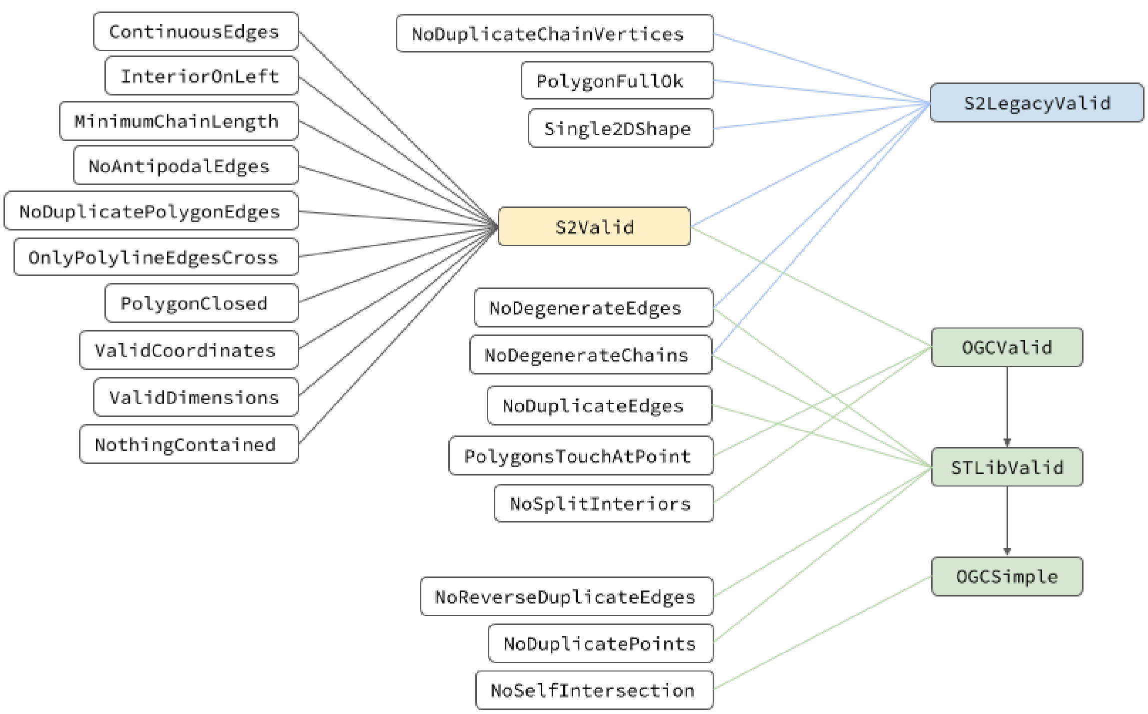
We define five semantic models to cover the background cases discussed above: S2Valid, S2LegacyValid, OGCValid, STLibValid, and OGCSimple.

S2Valid is the most lax semantic model and does basic checking that interiors are properly oriented, geometry doesn't overlap, etc. Degeneracies are explicitly supported. Altogether these are the requirements needed to be able to use S2BooleanOperation on the geometry.

S2LegacyValid is intended to support S2Polygon validation semantics, and is a combination of S2Valid, Single2DShape and other predicates.

Finally, we differentiate between geometry that is merely valid according to the OGC specification with OGCValid, we add additional constraints to check for dissolved geometry for STLibValid and, lastly, we check for polyline self intersection for OGCSimple.

These rules are shown in the following dependency graph showing how predicates roll up into final semantic categories:



Index Confidence

Given a pointer to an S2ShapeIndex, how do we know that it contains consistent information? It's possible that a user could give us an implementation that's inaccurate in every way about the underlying geometry (inconsistent interior state, etc).

It's possible to bootstrap confidence in an index, but it requires repeating the interior tracking logic that would have been used to build it and verifying that it's consistent. This is somewhat expensive and, in general, we don't expect to be dealing with adversarial indices.

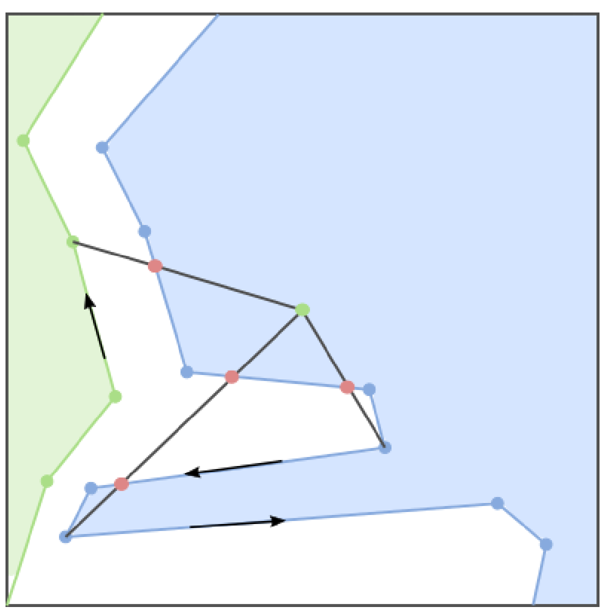
So we'll take the position for purposes of *geometry* validation that indexes were constructed properly (e.g. via MutableS2ShapeIndex or similar), so that the relationship between edges and cell centers is *consistent* and edges are in their proper cells.

Interiors On The Left

As long as we don't have any duplicate edges, we can *assume* that the interior state of the index is correct and check edge orientation independently on a cell-by cell basis. If the interior state *isn't* correct, then we'll return an error that edges aren't properly oriented.

We have to be a bit careful, however, since we can have multiple edges of a single shape in a cell, and they are not, in general, oriented the same with respect to the cell center. We can use S2EdgeCrosser to count the number of crossings with other edges from the same polygon between one edge vertex and the cell center. The containment state of the cell ***xor*** each crossing will tell us the orientation the edge should have.

We can see below that edges that should be oriented CCW with respect to the cell center will have an even number of crossings and those that should be CW have an odd number of crossings.



We know that every time we cross an edge, we toggle the interior, and that interiors are on the left. So, every time we cross an edge of the same polygon, the orientation of edges relative to the cell center must flip to maintain consistency.

We also assume that edges and cell centers are consistent, *but* it's possible that they can be consistent, but in the wrong direction. The interior tracking for a polygon is either entirely correct or entirely wrong. But, since we assume consistency, we only have to check a shape's chains once the first time we encounter it to convince ourselves that all the interior flags for that chain are correct.

Initial Seeding

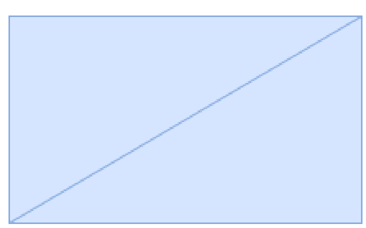
So, we can draw an imaginary line from a vertex to the cell center, toggling as we go, but what do we use for the initial interior state at the vertex? Since the vertex is part of the shape boundary is it contained or not? And we may have multiple edges incident on the vertex.

To resolve this we can note one property that's very useful: The interior of a polygon is always on the left of an edge. That means if we start at an edge and rotate an infinitesimal amount counter-clockwise around its first vertex, we'll always be in the interior of the shape, from the edge's perspective. If we keep rotating, any further edges must toggle the interior or they're improperly oriented (see above).

Given that, we can take all the edges incident on the vertex we want to test, sort them counter-clockwise, and toggle the interior bit on each one we hit until we hit a synthetic target edge we draw from the vertex to the cell center. This gives us the correct starting state and we can proceed to cross edges and toggle the interior normally.

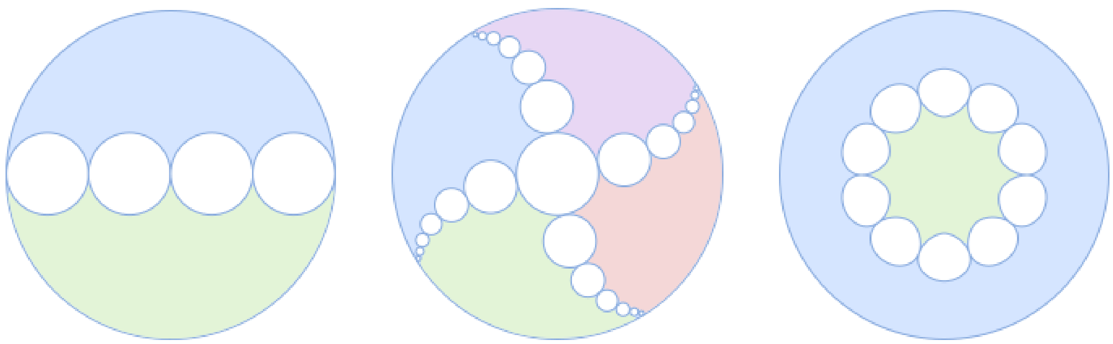
Connected Interiors

It's possible to have edges that split interiors without any interior vertices:

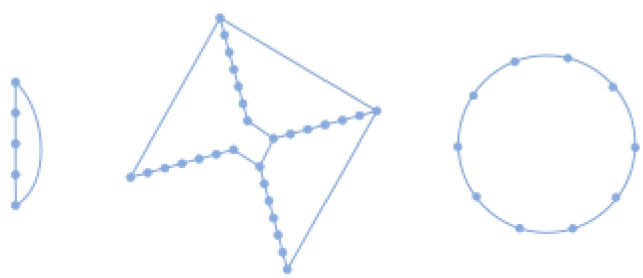


There's no containment check that will notice that the diagonal edge is in the interior of the polygon, nor will edge-crossing tests save us. Fortunately, there's no way to add an edge in this manner that won't make the interior state inconsistent, and thus either the edges in the top half or the bottom half will not be oriented properly.

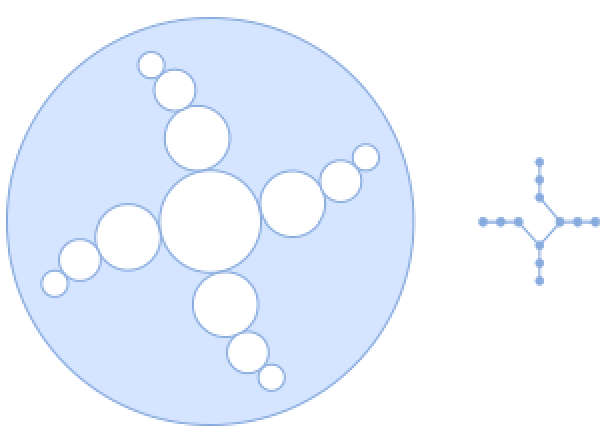
In general, split interiors are allowed as long as it's not due to an aberrant edge like above, or caused by self-intersection, S2BooleanOperation is fully capable of handling multiple interior areas. OGC semantics, however, explicitly require that polygon interiors be connected. And, in general, interiors can be split in arbitrarily complex ways by chaining holes together, including cases where the holes are disjoint from the shell:



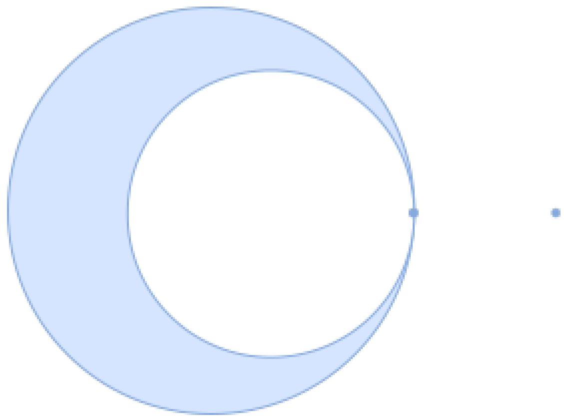
We thus need a robust way of detecting chains that touch and working backwards to determine whether they split the interior or not. We can reframe the problem in graph language by considering each point in shape where more than two edges meet (a *tangent point)* to represent a node, and each path of edges along a chain between tangents to be an edge in the graph. If we do this, the three shapes above look like the following graphs:



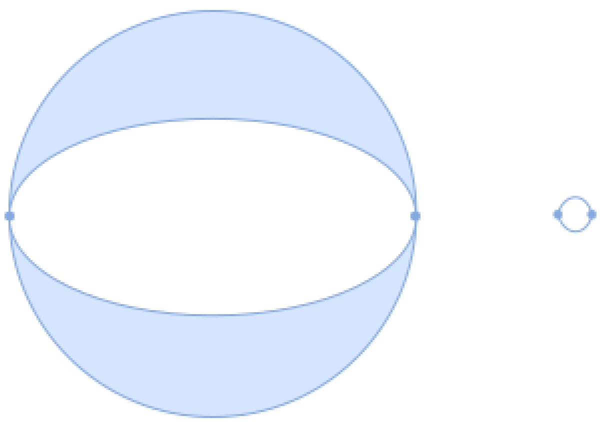
Note that we do *not* completely connect tangent points on a given chain back to the start, as that would always complete a cycle in the graph along a chain, and isn't necessary to capture the connectivity of the chains. If we removed the outer three holes in each arm from the second example, the graph would become properly acyclic since the centermost hole does not form a cycle:



If we consider any shape where the chains never touch, we can see immediately that it will be a null graph containing no nodes or edges, and thus trivially non-cyclic. In the case of a polygon with one interior ring that touches at *one* tangent point. The graph would contain one node and no edges, again trivially non-cyclic:



If we allow the hole to touch the other side and split the interior, we have two nodes and two edges (one along the shell and one along the hole), a cyclic graph:

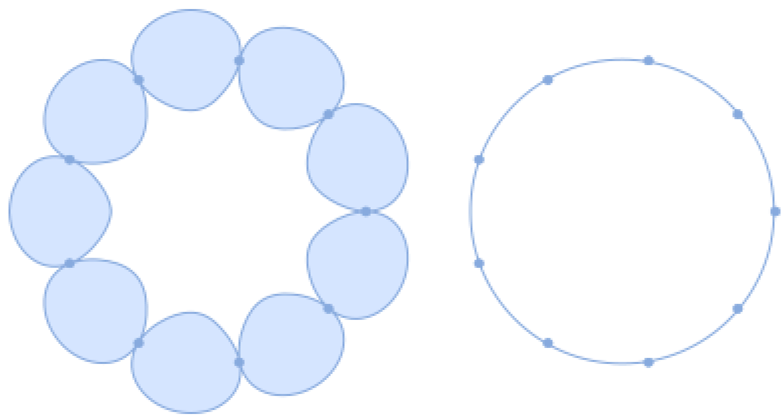


To split the interior of a polygon, we need to form two interior regions that can't be continuously reached from each other. A useful analogy for deciding whether two regions are connected is "Can I put a pen down and draw from one region into the other without crossing the polygon boundary?". If the answer is yes, then the regions are connected, otherwise they're not.

The only way an interior can become split is by forming a closed loop around an interior region, either by connecting at least two points on the shell by a chain of holes, or by drawing a connected chain of holes in the interior that separates a region.

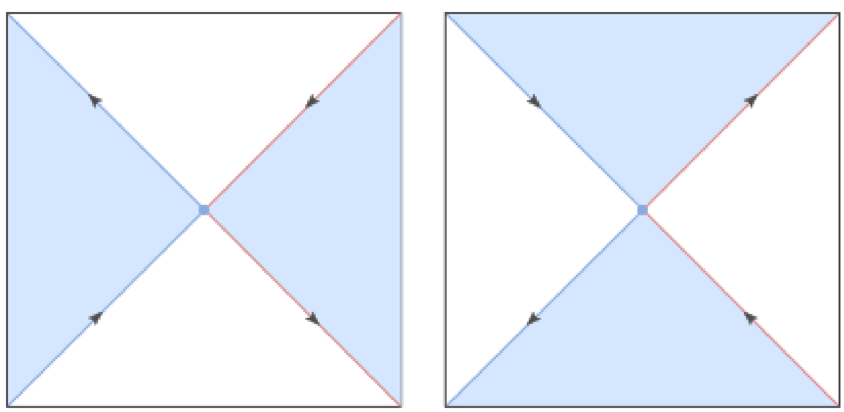
Either way, there are points on the polygon boundary (interior or exterior) that you can start at, and, moving along the boundary of the polygon, return to your starting point. Polygons with disconnected interiors have *cycles*.

But, is the converse true? If a graph has cycles does it represent a shape with a split interior? The answer, in general, is no. Without even resorting to the fact that S2 works on a closed 2D surface, we can find a polygon with a cyclic graph that doesn't have a split interior, which is simply the complement of the third polygon example above:



This looks exactly like the graph for the complement polygon above. We'd like to exclude these valid polygons from our test, but how can we tell them apart? Let's further consider how chains might meet at a point.

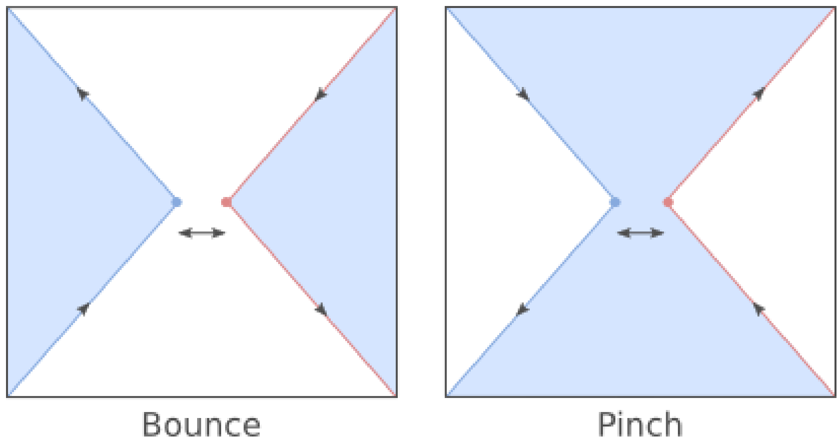
Since we're already going to check for crossing edges, and properly oriented interiors, there's only two ways that two chains can meet at a tangent point: the interior can be between edges of the same chain, or between edges of different chains. We can call these two situations a *bounce* or a *pinch*, respectively:



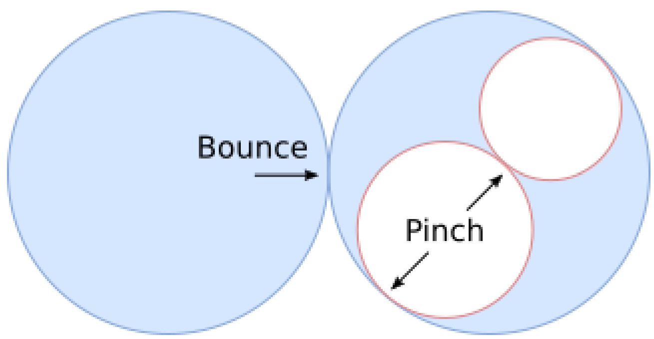
Bounce Pinch

We can see in the first example how each incoming chain “bounces” off the intersection point, and doesn't rely on the other chain to define the interior; if we pulled the chains apart horizontally we'd have two disjoint chains each containing the interior.

In the second example, however, each incoming edge in each chain relies on an outgoing edge from the *other* chain to define the interior. If we pulled *these* chains apart horizontally, the interior would stretch with them:



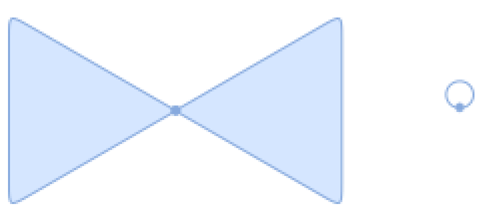
We can see how the “pinch” configuration in some sense separates the interior in a continuous topological way. A bounce is equivalent to two shells touching at a tangent point, while a pinch is equivalent to two holes touching, *or* a hole touching a shell.



We can easily detect pinches by looking at each ingoing edge at a vertex, and seeing if the next edge (ordered CW around the vertex) is the next edge in the same chain or not. This makes intuitive sense because we’re effectively saying that, for a bounce, there can’t be any intervening edges between the ingoing and outgoing edges of a particular chain.

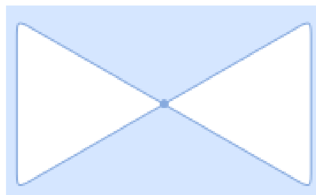
If we restrict ourselves to building a graph with nodes just for pinch points, then we can detect split interiors by detecting cycles in the graph. There are a few special cases we have to think about though.

The first is a shell that’s pinched through a self tangency:

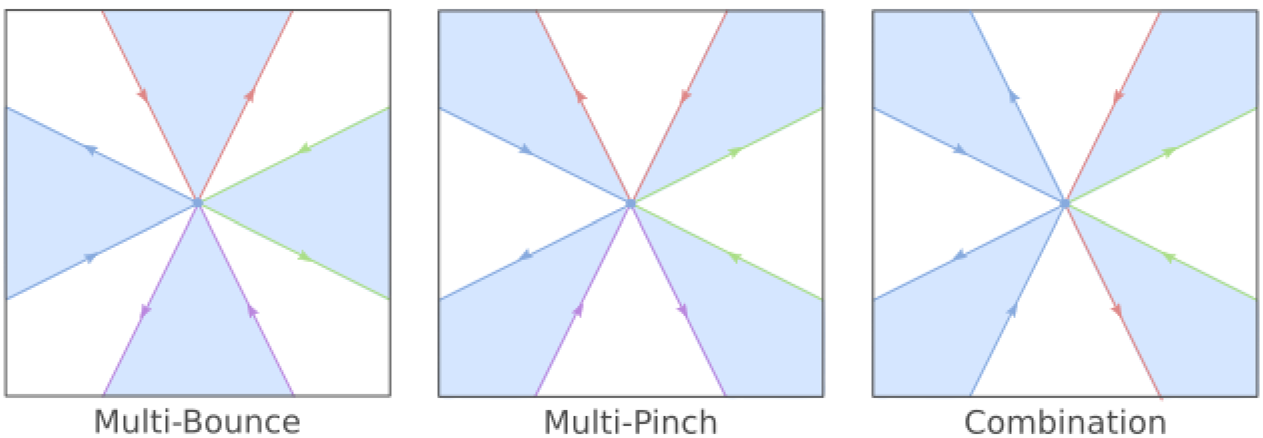


This is still equivalent to a pinch, and splits the interior. It's somewhat special in that the two chains that are pinching are one and the same, but, we can treat it the same as a pinch between any two chains, we'll just add the same pinch point *twice* for the same chain. This will ensure a cycle in the resulting graph, which we'll detect.

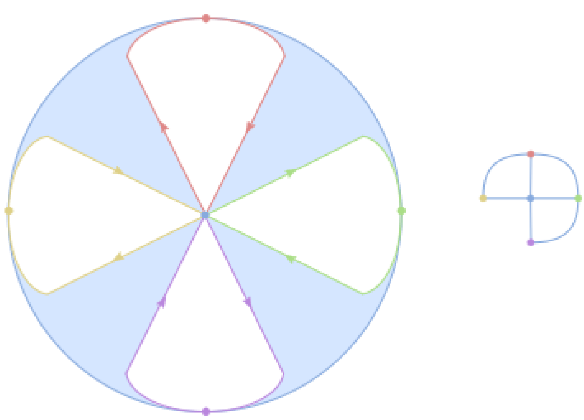
In the case of a hole that's self tangent, the pinch becomes a bounce, and we'll safely ignore it:



Next, there's no reason that a tangency has to be between only two chains, for example, here's a bounce and pinch configuration between four chains, we can even have a combination of bounces and pinches:



If we look at a shape containing a pinch like that, and its associated graph:



We can see why we were careful to define the graph in terms of the tangent points, with edges connecting them. The central point shared by the holes on its own doesn't generate a cycle. If we disconnected the holes from the shell, the graph would collapse to a single node, which is trivially acyclic.

We can gather the information we need for the graph as we work through the index. If we store a mapping of **S2Shape* => [(chain, vertex)]** pairs for each shape that has tangency points, then at the end of the validation process we can use a disjoint set data structure to find cycles[7] as follows:

```
def CheckInteriorConnected(tangent_list):
  tangent_list = sort(tangent_list by chain) # exact ordering within chain is irrelevant

  ds = DisjointSet()

  # Create singleton sets for each point
  for value in tangent_list:
    ds.Make(value.vertex)

  # Add edges, if we find two vertices connected, we have a loop
  for ii < len(tangent_list)-1:
     curr = tangent_list[ii]
     next = tangent_list[ii+1]

     if (curr.chain == next.chain):
        curr_root = ds.Find(curr.vertex)
        next_root = ds.Find(next.vertex)
        if (curr_root == next_root):
           return Error("Disconnected interior found")
       ds.Union(curr_root, next_root)

  return true
```

Note that we expect most shapes to have zero tangent points so in the common case we do no work (O(0)!). In the more general case, the disjoint-set has ~O(1) complexity for Find and Union (technically O(Ackermann^{-1}(n)) which is < 5 for all realistic values), so processing the edges is O(n) and thus the overhead is dominated by the sort call which is O(n log n) in the number of tangent points.

[7] See tutorial here

Validation Process Details

With those tools available, we're now ready to define the validation processing. We can break the process up into two stages. First, we can run any checks that can be done independently on the shapes in the index. These include:

- ValidDimensions
- MinimumChainLength
- PolygonClosed
- Single2DShape
- PolygonFullOk

These are easy O(1) checks that don't require index information to verify. Checking them first will at minimum tell us that polygons in the index represent some closed (possibly degenerate) loop, and they provide easy criterion for short-circuiting the rest of the validation.

Now, we can begin the validation process in earnest. As we go we'll maintain two additional pieces of state: a map from S2Shape to the CellId where we first encountered it, and map from S2Shape to a list of (chain,vertex) pairs for separated interior tracking. We'll proceed cell-by-cell in the index, validating as we go. We'll assume that the index is valid until proven otherwise.

The following pseudocode shows the basic logic:

```
# Initial O(1) checks.
MaybeCheck(Single2DShape(index))
for shape in index:
  Check(ValidDimensions(shape))
  if (shape.dimension() == 2):
    Check(MinimumChainLength(shape))
    Check(PolygonClosed(shape))
    MaybeCheck(PolygonFullOk(shape))

# Store tangency points
tangent_map = Map<S2Shape*,[(chain,vertex)]>()

for cell in index:
  # Check dimension 2 edges in the cell for duplicates.  By definition duplicate
  # edges will appear in the same cell(s).
  Check(NoDuplicatePolygonEdges(cell))
  MaybeCheck(NoReverseDuplicateEdges(cell))

  # If a chain touches itself, both edges will end in the same cell at some point, so
  # we can detect duplicate edges locally by just checking the edges in each cell.
  MaybeCheck(NoDuplicateChainVertices(cell))

  # Maybe we shouldn't have duplicate edges at all (e.g. for OGCValid)
  MaybeCheck(NoDuplicateEdges(cell))

  # Maybe check any points in the cell to ensure they don't overlap any other
  # point or vertex.
```

```
  MaybeCheck(NoDuplicatePoints(cell))

  # Check basic validity for edges (doesn't affect duplicate check).
  for edge in cell:
    Check(ValidCoordinates(edge))
    MaybeCheck(NotAntipodal(edge))
    MaybeCheck(NotDegenerate(edge))

  # Check that polygons don’t have crossing edges.
  Check(OnlyPolylineEdgesCross(cell))
  MaybeCheck(NoChainCrossings(cell))

  # Disallow degenerate chains for STLib
  MaybeCheck(NoDegenerateChains(cell))

  # Find any pinch points and add edges
  for shape in cell:
    tangent_map[shape].append(TangentPoints(shape, cell))

  for edge in cell:
    # Check that edges are continuous.
    if (edge.shape.dimension == 1 or 2):
       Check(edge.v1 == next(edge).v0)

    # Count crossings and check that edges are oriented to keep the interior on the left.
    if (edge.shape.dimension == 2):
       Check(EdgeOrientedProperly(edge))

    # Since there's no crossing edges, we know each edge is either entirely inside,
    # entirely outside, or a vertex of each polygon, so we can verify NothingContained by
    # checking the first vertex of each edge against each polygon.
    for shape in cell:
       if not shape in first_cells:
         first_cells[shape] = cell

       if (edge.shape != shape):
         <draw line from cell center to edge.v0, toggle shape interior bit>
         Check(!inside(shape))

# Check for disjoint interiors.
for shape, tangent_list in tangent_map:
  CheckInteriorConnected(tangent_list)

# Index is now valid.
```