The Atomic Theory as Applied To Gases, with Some Experiments on the Viscosity of Air

by

Silas W. Holman

Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of

BACHELOR OF SCIENCE IN PHYSICS

at the

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ABSTRACT

The developments of the "kinetic theory" of gases made within the last ten years have enabled it to account satisfactorily for many of the laws of gases. The mathematical deductions of Clausius, Maxwell and others, based upon the hypothesis of a gas composed of molecules acting upon each other at impact like perfectly elastic spheres, have furnished expressions for the laws of its elasticity, viscosity, conductivity for heat, diffusive power and other properties. For some of these laws we have experimental data of value in testing the validity of these deductions and assumptions. Next to the elasticity, perhaps the phenomena of the viscosity of gases are best adapted to investigation.[1]

Thesis supervisor: Edward C. Pickering
Title: Professor of Physics

[1]Text from Holman (1876): doi:10.2307/25138434.

Acknowledgments

Write your acknowledgments here.

Biographical Sketch

Silas Whitcomb Holman was born in Harvard, Massachusetts on January 20, 1856. He received his S.B. degree in Physics from MIT in 1876, and then joined the MIT Department of Physics as an Assistant. He became Instructor in Physics in 1880, Assistant Professor in 1882, Associate Professor in 1885, and Full Professor in 1893. Throughout this period, he struggled with increasingly severe rheumatoid arthritis. At length, he was defeated, becoming Professor Emeritus in 1897 and dying on April 1, 1900.

Holman's light burned brilliantly before his tragic and untimely death. He published extensively in thermal physics, and authored textbooks on precision measurement, fundamental mechanics, and other subjects. He established the original Heat Measurements Laboratory. Holman was a much admired teacher among both his students and his colleagues. The reports of his department and of the Institute itself refer to him frequently in the 1880's and 1890's, in tones that gradually shift from the greatest respect to the deepest sympathy.

Holman was a student of Professor Edward C. Pickering, then head of the Physics department. Holman himself became second in command of Physics, under Professor Charles R. Cross, some years later. Among Holman's students, several went on to distinguish themselves, including: the astronomer George E. Hale ('90) who organized the Yerkes and Mt. Wilson observatories and who designed the 200 inch telescope on Mt. Palomar; Charles G. Abbot ('94), also an astrophysicist and later Secretary of the Smithsonian Institution; and George K. Burgess ('96), later Director of the Bureau of Standards.

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Chapter 1

Introduction

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1.1 A section discussing the first issue: J/ψ

We begin with some ideas from the literature [6,7].

$$\frac{\partial}{\partial t}\left[\rho\left(e+|\vec{u}|^2/2\right)\right]+\nabla\cdot\left[\rho\left(h+|\vec{u}|^2/2\right)\vec{u}\right]=-\nabla\cdot\vec{q}+\rho\vec{u}\cdot\vec{g}+\frac{\partial}{\partial x_j}\left(d_{ji}u_i\right) \tag{1.1}$$

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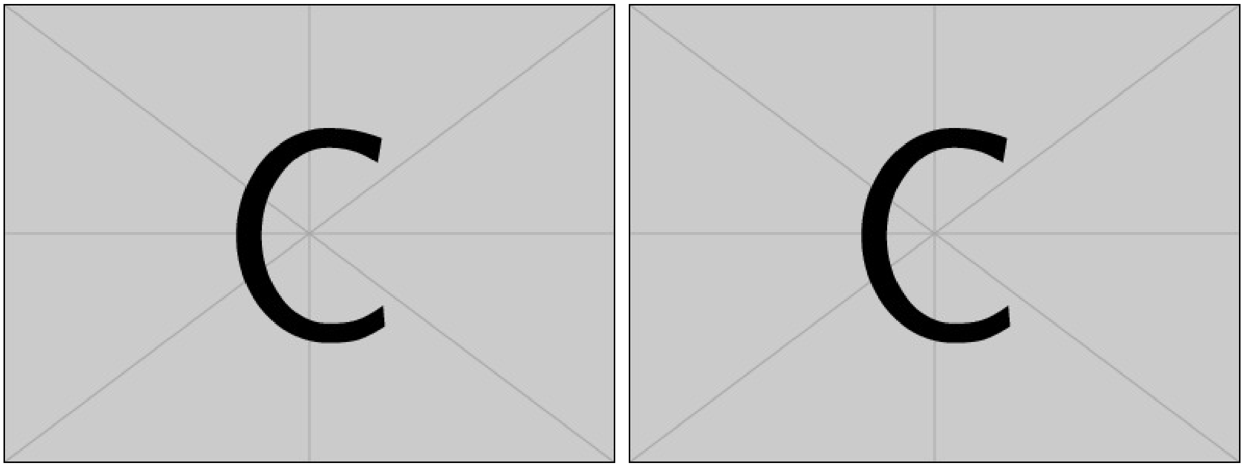

(a) First subfigure (b) Second subfigure

Figure 1.1: A figure with two subfigures: (a) first subfigure; (b) second subfigure.

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1.1.1 Subsection eqn. (1.2)

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A subsubsection

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$$L(\mathbf{A}) = \begin{pmatrix} \frac{\varphi}{(\varphi_1,\varepsilon_1)} & 0 & \dots & \dots & \dots & 0 \\ \frac{\varphi k_{2,1}}{(\varphi_2,\varepsilon_1)} & \frac{\varphi}{(\varphi_2,\varepsilon_2)} & 0 & \dots & \dots & 0 \\ \frac{\varphi k_{3,1}}{(\varphi_3,\varepsilon_1)} & \frac{\varphi k_{3,2}}{(\varphi_3,\varepsilon_2)} & \frac{\varphi}{(\varphi_3,\varepsilon_3)} & 0 & \dots & 0 \\ \vdots & \vdots & & \ddots & & \vdots \\ \frac{\varphi k_{n-1,1}}{(\varphi_{n-1},\varepsilon_1)} & \frac{\varphi k_{n-1,2}}{(\varphi_{n-1},\varepsilon_2)} & \dots & \frac{\varphi k_{n-1,n-2}}{(\varphi_{n-1},\varepsilon_{n-2})} & \frac{\varphi}{(\varphi_{n-1},\varepsilon_{n-1})} & 0 \\ \frac{\varphi k_{n,1}}{(\varphi_n,\varepsilon_1)} & \frac{\varphi k_{n,2}}{(\varphi_n,\varepsilon_2)} & \dots & \dots & \frac{\varphi k_{n,n-1}}{(\varphi_n,\varepsilon_{n-1})} & \frac{\varphi}{(\varphi_n,\varepsilon_n)} \end{pmatrix} \tag{1.2}$$

1.2 Description our paradigm

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1.2.1 Conversion to a metaheuristic

Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetuer eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce

[1] First footnote. $a_h = F_m$ See section 1.4.
[2] Another interesting detail.
[3] And another really important idea to have in mind [12–17].

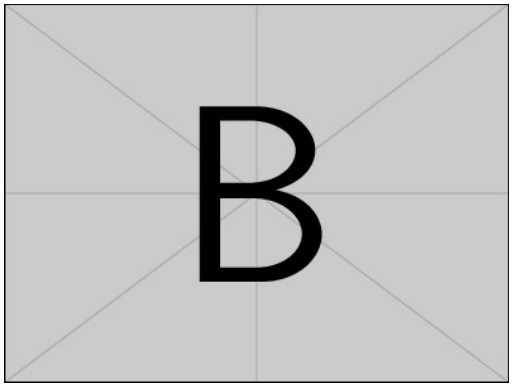

Figure 1.2: Caption text [8].

blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor.

Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id, nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas, pede pede pretium lorem, quis consectetuer tortor sapien facilisis magna. Mauris quis magna varius nulla scelerisque imperdiet. Aliquam non quam. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo. This concept is discussed further in section 1.4, and Refs. [18,19].

1.3 Other generalizations

1.3.1 The most general case

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus. And another citation, so that our sources will be unambiguous [20].

$$x\,\mathrm{Na(NH_4)HPO_4} \xrightarrow{\Delta} \mathrm{(NaPO_3)}_x + x\,\mathrm{NH_3}\uparrow + x\,\mathrm{H_2O} \tag{1.3}$$

$$^{234}_{90}\mathrm{Th} \longrightarrow {}^{0}_{-1}\beta + {}^{234}_{91}\mathrm{Pa} \tag{1.4}$$

$$\mathrm{SO_4}^{2-} + \mathrm{Ba}^{2+} \longrightarrow \mathrm{BaSO_4}\downarrow \tag{1.5}$$

$$\mathrm{Zn}^{2+} \underset{+2\,\mathrm{H}^+}{\overset{+2\,\mathrm{OH}^-}{\rightleftharpoons}} \underset{\text{amphoteric hydroxide}}{\mathrm{Zn(OH)_2}\downarrow} \underset{+2\,\mathrm{H}^+}{\overset{+2\,\mathrm{OH}^-}{\rightleftharpoons}} \underset{\text{tetrahydroxozincate}}{[\mathrm{Zn(OH)_4}]^{2-}} \tag{1.6}$$

These examples of chemical formulæ are copied directly from the documentation of the `mhchem` package, which was used to typeset them.

1.4 Baroclinic generation of vorticity

Substitution of the particle acceleration and application Stokes theorem leads to the *Kelvin-Bjerknes circulation theorem*, for $\rho \neq \mathrm{fn}(p)$:

$$\frac{d\Gamma}{dt} = \frac{d}{dt}\int_C \mathbf{u}\cdot d\mathbf{r} \tag{1.7}$$

$$= \int_C \frac{D\mathbf{u}}{Dt}\cdot d\mathbf{r} + \underbrace{\int_C \mathbf{u}\cdot d\left(\frac{d\mathbf{r}}{dt}\right)}_{=0} \tag{1.8}$$

$$= \iint_S \nabla\times\frac{D\mathbf{u}}{Dt}\cdot d\mathbf{A} \tag{1.9}$$

$$= \iint_S \nabla p\times\nabla\left(\frac{1}{\rho}\right)\cdot d\mathbf{A} \tag{1.10}$$

Baroclinic generation of vorticity accounts for the sea breeze and various other atmospheric currents in which temperature, rather than pressure, creates density gradients. Further, this phenomenon accounts for ocean currents in straits joining more and less saline seas, with surface currents flowing from the fresher to the saltier water and with bottom current going oppositely.

Nomenclature for Chapter 1

Roman letters

C	material curve
$\mathbf{r}$	material position [m]
$\mathbf{u}$	velocity [m s^{-1}]

Greek letters

Γ	circulation [m^2 s^{-1}]
ρ	mass density [kg m^{-3}]
$\boldsymbol{\omega}$	vorticity [s^{-1}]

Table 1.1: The error function and complementary error function

x	$\mathrm{erf}(x)$	$\mathrm{erf}(x)$	x	$\mathrm{erfc}(x)$	$\mathrm{erfc}(x)$
0.00	0.00000	1.00000	1.10	0.88021	0.11980
0.05	0.05637	0.94363	1.20	0.91031	0.08969
0.10	0.11246	0.88754	1.30	0.93401	0.06599
0.15	0.16800	0.83200	1.40	0.95229	0.04771
0.20	0.22270	0.77730	1.50	0.96611	0.03389
0.30	0.32863	0.67137	1.60	0.97635	0.02365
0.40	0.42839	0.57161	1.70	0.98379	0.01621
0.50	0.52050	0.47950	1.80	0.98909	0.01091
0.60	0.60386	0.39614	1.8214	0.99000	0.01000
0.70	0.67780	0.32220	1.90	0.99279	0.00721
0.80	0.74210	0.25790	2.00	0.99532	0.00468
0.90	0.79691	0.20309	2.50	0.99959	0.00041
1.00	0.84270	0.15730	3.00	0.99998	0.00002

Appendix A

Code listing

This example uses the `listings` package.

```
function print_rate(kappa,xMin,xMax,npoints,option)
    local c = 1-kappa*kappa
    local croot = (1-kappa*kappa)^(1/2)
    local logx = math.log(xMin)
    local psi = 0

    local xstep = (math.log(xMax)-math.log(xMin))/(npoints-1)

    arg0 = math.sqrt(xMin/c)
    psi0 = (1/c)*math.exp((kappa*arg0)^2)*(erfc(kappa*arg0)-erfc(arg0))

    if option~=[[]] then
      tex.sprint("\\addplot+["..option.."] coordinates{")
      -- addplot+ for color cycle to work
    else
      tex.sprint("\\addplot+ coordinates{")
    end
    tex.sprint("("..xMin..","..psi0..")")

    for i=1, (npoints-1) do
      x = math.exp(logx + xstep)
      arg = math.sqrt(x/c)
      karg = kappa*arg
      if karg<5 then
    -- this break compensates for exp(karg^2), which multiplies the error in the erf approximation...
        logpsi = -math.log(croot) + karg^2 + math.log(erfc(karg)-erfc(arg))
        psi = math.exp(logpsi)
      else
```

```
            psi = (1/(karg) - 1/(2*(karg^3)) + 3/(4*(arg^5)) )/(1
                .77245385*croot)
            -- this is the large x asymptote of the reaction rate
        end
        logx = math.log(x)
        tex.sprint("("..x..","..psi..")")
      end
      tex.sprint("}")
end
\end{luacode*}
```

Appendix B

One-term coefficients for heat conduction

B.1 A multipage table of numbers

This example uses the `longtable` package: $\theta = A_1 f_1 \exp(-\lambda_1^2 \mathrm{Fo})$, $\overline{\theta} = D_1 \exp(-\lambda_1^2 \mathrm{Fo})$.

Table B.1: One-term coefficients for one-dimensional heat conduction with a convective boundary condition. Data follow H. D. Baehr and K. Stephan [21].

Bi	*Plate*			*Cylinder*			*Sphere*		
	λ_1	A_1	D_1	λ_1	A_1	D_1	λ_1	A_1	D_1
0.01	0.09983	1.0017	1.0000	0.14124	1.0025	1.0000	0.17303	1.0030	1.0000
0.02	0.14095	1.0033	1.0000	0.19950	1.0050	1.0000	0.24446	1.0060	1.0000
0.03	0.17234	1.0049	1.0000	0.24403	1.0075	1.0000	0.29910	1.0090	1.0000
0.04	0.19868	1.0066	1.0000	0.28143	1.0099	1.0000	0.34503	1.0120	1.0000
0.05	0.22176	1.0082	0.9999	0.31426	1.0124	0.9999	0.38537	1.0150	1.0000
0.06	0.24253	1.0098	0.9999	0.34383	1.0148	0.9999	0.42173	1.0179	0.9999
0.07	0.26153	1.0114	0.9999	0.37092	1.0173	0.9999	0.45506	1.0209	0.9999
0.08	0.27913	1.0130	0.9999	0.39603	1.0197	0.9999	0.48600	1.0239	0.9999
0.09	0.29557	1.0145	0.9998	0.41954	1.0222	0.9998	0.51497	1.0268	0.9999
0.10	0.31105	1.0161	0.9998	0.44168	1.0246	0.9998	0.54228	1.0298	0.9998
0.15	0.37788	1.0237	0.9995	0.53761	1.0365	0.9995	0.66086	1.0445	0.9996
0.20	0.43284	1.0311	0.9992	0.61697	1.0483	0.9992	0.75931	1.0592	0.9993
0.25	0.48009	1.0382	0.9988	0.68559	1.0598	0.9988	0.84473	1.0737	0.9990
0.30	0.52179	1.0450	0.9983	0.74646	1.0712	0.9983	0.92079	1.0880	0.9985
0.40	0.59324	1.0580	0.9971	0.85158	1.0931	0.9970	1.05279	1.1164	0.9974
0.50	0.65327	1.0701	0.9956	0.94077	1.1143	0.9954	1.16556	1.1441	0.9960
0.60	0.70507	1.0814	0.9940	1.01844	1.1345	0.9936	1.26440	1.1713	0.9944
0.70	0.75056	1.0918	0.9922	1.08725	1.1539	0.9916	1.35252	1.1978	0.9925
0.80	0.79103	1.1016	0.9903	1.14897	1.1724	0.9893	1.43203	1.2236	0.9904
0.90	0.82740	1.1107	0.9882	1.20484	1.1902	0.9869	1.50442	1.2488	0.9880
1.00	0.86033	1.1191	0.9861	1.25578	1.2071	0.9843	1.57080	1.2732	0.9855
1.10	0.89035	1.1270	0.9839	1.30251	1.2232	0.9815	1.63199	1.2970	0.9828
1.20	0.91785	1.1344	0.9817	1.34558	1.2387	0.9787	1.68868	1.3201	0.9800
1.30	0.94316	1.1412	0.9794	1.38543	1.2533	0.9757	1.74140	1.3424	0.9770
1.40	0.96655	1.1477	0.9771	1.42246	1.2673	0.9727	1.79058	1.3640	0.9739
1.50	0.98824	1.1537	0.9748	1.45695	1.2807	0.9696	1.83660	1.3850	0.9707

Table B.1: (continued)

Bi	*Plate*			*Cylinder*			*Sphere*		
	λ_1	A_1	D_1	λ_1	A_1	D_1	λ_1	A_1	D_1
1.60	1.00842	1.1593	0.9726	1.48917	1.2934	0.9665	1.87976	1.4052	0.9674
1.70	1.02725	1.1645	0.9703	1.51936	1.3055	0.9633	1.92035	1.4247	0.9640
1.80	1.04486	1.1695	0.9680	1.54769	1.3170	0.9601	1.95857	1.4436	0.9605
1.90	1.06136	1.1741	0.9658	1.57434	1.3279	0.9569	1.99465	1.4618	0.9570
2.00	1.07687	1.1785	0.9635	1.59945	1.3384	0.9537	2.02876	1.4793	0.9534
2.20	1.10524	1.1864	0.9592	1.64557	1.3578	0.9472	2.09166	1.5125	0.9462
2.40	1.13056	1.1934	0.9549	1.68691	1.3754	0.9408	2.14834	1.5433	0.9389
2.60	1.15330	1.1997	0.9509	1.72418	1.3914	0.9345	2.19967	1.5718	0.9316
2.80	1.17383	1.2052	0.9469	1.75794	1.4059	0.9284	2.24633	1.5982	0.9243
3.00	1.19246	1.2102	0.9431	1.78866	1.4191	0.9224	2.28893	1.6227	0.9171
3.50	1.23227	1.2206	0.9343	1.85449	1.4473	0.9081	2.38064	1.6761	0.8995
4.00	1.26459	1.2287	0.9264	1.90808	1.4698	0.8950	2.45564	1.7202	0.8830
4.50	1.29134	1.2351	0.9193	1.95248	1.4880	0.8830	2.51795	1.7567	0.8675
5.00	1.31384	1.2402	0.9130	1.98981	1.5029	0.8721	2.57043	1.7870	0.8533
6.00	1.34955	1.2479	0.9021	2.04901	1.5253	0.8532	2.65366	1.8338	0.8281
7.00	1.37662	1.2532	0.8932	2.09373	1.5411	0.8375	2.71646	1.8673	0.8069
8.00	1.39782	1.2570	0.8858	2.12864	1.5526	0.8244	2.76536	1.8920	0.7889
9.00	1.41487	1.2598	0.8796	2.15661	1.5611	0.8133	2.80443	1.9106	0.7737
10.00	1.42887	1.2620	0.8743	2.17950	1.5677	0.8039	2.83630	1.9249	0.7607
12.00	1.45050	1.2650	0.8658	2.21468	1.5769	0.7887	2.88509	1.9450	0.7397
14.00	1.46643	1.2669	0.8592	2.24044	1.5828	0.7770	2.92060	1.9581	0.7236
16.00	1.47864	1.2683	0.8541	2.26008	1.5869	0.7678	2.94756	1.9670	0.7109
18.00	1.48830	1.2692	0.8499	2.27556	1.5898	0.7603	2.96871	1.9734	0.7007
20.00	1.49613	1.2699	0.8464	2.28805	1.5919	0.7542	2.98572	1.9781	0.6922
25.00	1.51045	1.2710	0.8400	2.31080	1.5954	0.7427	3.01656	1.9856	0.6766
30.00	1.52017	1.2717	0.8355	2.32614	1.5973	0.7348	3.03724	1.9898	0.6658
35.00	1.52719	1.2721	0.8322	2.33719	1.5985	0.7290	3.05207	1.9924	0.6579
40.00	1.53250	1.2723	0.8296	2.34552	1.5993	0.7246	3.06321	1.9942	0.6519
50.00	1.54001	1.2727	0.8260	2.35724	1.6002	0.7183	3.07884	1.9962	0.6434
60.00	1.54505	1.2728	0.8235	2.36510	1.6007	0.7140	3.08928	1.9974	0.6376
80.00	1.55141	1.2730	0.8204	2.37496	1.6013	0.7085	3.10234	1.9985	0.6303
100.00	1.55525	1.2731	0.8185	2.38090	1.6015	0.7052	3.11019	1.9990	0.6259
200.00	1.56298	1.2732	0.8146	2.39283	1.6019	0.6985	3.12589	1.9998	0.6170
∞	1.57080	1.2732	0.8106	2.40483	1.6020	0.6917	3.14159	2.0000	0.6079

References

[1] D. K. Edwards. "Radiative Transfer Characteristics of Materials." *ASME J. Heat Transfer*, **91**(1), Feb. 1969, pp. 1–15. DOI: 10.1115/1.3580108.

[2] E. T. Whittaker and G. N. Watson. *A Course of Modern Analysis*. 3rd ed. Cambridge, UK: Cambridge University Press, 1920. URL: https://archive.org/details/courseofmodernan00whit.

[3] J. T. Kirk. *Decline and Fall of the Romulan Empire*. 7th ed. Humankind's Greatest Writings 23. T'Paal: Vulcan Free Press, 2288. To appear.

[4] W. S. Churchill. *The Gathering Storm*. In: *The Second World War*. Vol. 1. Boston: Houghton Mifflin Co., 1948. Chap. 5, "The Locust Years", pp. 66–89. URL: https://www.worldcat.org/oclc/3025315.

[5] J. W. Gibbs. "On the Form of the Teeth of Wheels in Spur Gearing." Ph.D. dissertation. New Haven, CT: Yale University, 1863.

[6] C. Fong. *Analytical Methods for Squaring the Disc*. *ArXiv e-prints*. Sept. 2015. arXiv: 1509.06344.

[7] M. Sharpe. *New TX Font Package*. Version 1.71. Comprehensive TEX Archive Network, Mar. 2022. URL: https://ctan.org/pkg/newtx (visited on 05/29/2023).

[8] M. Galassi, J. Davies, J. Theiler, B. Gough, G. Jungman, P. Alken, M. Booth, F. Rossi, and R. Ulerich. *GNU Scientific Library*. Version 2.4. Free Software Foundation. Boston, MA, 2017. URL: https://www.gnu.org/software/gsl/ (visited on 12/12/2018).

[9] J. Swaminathan, R. L. Stover, E. W. Tow, D. M. Warsinger, and J. H. Lienhard. "Effect of Practical Losses on Optimal Design of Batch RO Systems." In: *Proceedings of IDA World Congress on Desalination and Water Reuse* (São Paulo, Brazil, Oct. 15–20, 2017). IDA17WC-58334. International Desalination Association, Oct. 2017. HDL: 1721.1/111971.

[10] F. W. J. Olver, A. B. Olde Daalhuis, D. W. Lozier, B. I. Schneider, R. F. Boisvert, C. W. Clark, B. R. Miller, and B. V. Saunders, eds. *NIST Digital Library of Mathematical Functions*. Version 1.1.19. Gaithersburg, MD: National Institute of Standards and Technology, Mar. 2023. URL: https://dlmf.nist.gov/ (visited on 05/29/2023).

[11] American Mathematical Society and The LATEX Project. *User's Guide for the amsmath Package*. Version 2.1. Comprehensive TEX Archive Network. Feb. 2020. URL: https://ctan.org/tex-archive/macros/latex/required/amsmath/amsldoc.pdf (visited on 05/29/2023).

[12] W. C. Reynolds, W. M. Kays, and S. J. Kline. *Heat Transfer in the Incompressible Turbulent Boundary Layer. I—Constant Wall Temperature*. NASA Technical Memorandum No. 12-1-58W. Washington, DC: National Aeronautics and Space Administration, Dec. 1958. HDL: 2060/19980228020.

[13] F. H. Clauser. “The Turbulent Boundary Layer.” In: *Advances in Applied Mechanics*. Ed. by H. L. Dryden and T. von Kármán. Vol. 4. Amsterdam, The Netherlands: Elsevier, 1956, pp. 1–51. DOI: 10.1016/S0065-2156(08)70370-3.

[14] J. H. Lienhard. “Heat Transfer in Flat-Plate Boundary Layers: A Correlation for Laminar, Transitional, and Turbulent Flow.” *ASME J. Heat Transfer*, **142**(6), 061805, June 2020. DOI: 10.1115/1.4046795.

[15] H. Johnson, ed. *Title of Edited Book*. New York, NY: John Wiley and Sons, Inc., 1980.

[16] H. W. Johnson, ed. *The Title of the Proceedings* (City, State, Apr. 27–28, 1965). Vol. 2. Organization. 1965. URL: https://news.mit.edu/2009/obit-johnson.

[17] S. van der Walt and N. Smith. *mpl Colormaps*. San Francisco, CA: GitHub, Sept. 2015. URL: https://bids.github.io/colormap/ (visited on 08/26/2018).

[18] L. Euler. “De Summis Serierum Reciprocarum.” *Commentarii Academiae Scientiarum Petropolitanae*, **7**(), 1740, pp. 123–134. arXiv: math/0506415. First communicated to Daniel Bernoulli in 1734 and read before the St. Petersburg Academy in December 1735.

[19] J. B. J. Fourier. *Théorie Analytique de la Chaleur*. Paris: Firmin Didot, Père et Fils, 1822. URL: https://archive.org/details/analyticaltheory00fourrich.

[20] J. I. Montijano, M. Pérez, L. Rández, and J. L. Varona. “Numerical Methods With LuaLaTeX.” *TUGboat*, **35**(1), Jan. 2014, pp. 51–56. URL: https://tug.org/TUGboat/tb35-1/tb109montijano.pdf.

[21] H. D. Baehr and K. Stephan. *Heat and Mass Transfer*. Berlin: Springer-Verlag, 1998. ISBN: 3-540-63695-1.