

# Package ‘ECMLE’

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**Title** Approximating Evidence via Bounded Harmonic Means

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**Description** Implements the Elliptical Covering Marginal Likelihood Estimator (ECMLE), a geometric method for approximating marginal likelihood from posterior draws and log-posterior evaluations. The method constructs a collection of non-overlapping ellipsoids in a high-posterior-density region, computes the covered volume, and combines this with posterior sample coverage to estimate model evidence. It is designed to stabilize harmonic-mean-based evidence approximation and can be applied in multimodal settings. The methodology is described in Naderi et al. (2025) <[doi:10.48550/arXiv.2510.20617](https://doi.org/10.48550/arXiv.2510.20617)>.

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ECMLE-package	<i>ECMLE: Elliptical Covering Marginal Likelihood Estimator</i>
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## Description

Implements Elliptical Covering Marginal Likelihood Estimator (ECMLE), a geometric method for approximating marginal likelihood from posterior draws and log-posterior evaluations. The approach constructs a collection of non-overlapping ellipsoids within a high-posterior-density region, computes the covered volume, and combines this with posterior sample coverage to estimate model evidence. The method is designed to stabilize harmonic-mean-based evidence approximation and can be applied in multimodal settings. The methodology is described in Naderi et al. (2025) <doi:10.48550/arXiv.2510.20617>.

## Details

The package provides the main estimator `ecmle()` with `print`, `summary`, and `plot` methods, visualisation functions `plot_ecmle_2d()` and `draw_ellipse_2d()`, a pair plot utility `pair_plot()`, and Rosenbrock (banana) posterior benchmark functions for testing and reproducible experiments.

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## See Also

[ecmle](#)

ecmle

*Estimate the log marginal likelihood with ECMLE***Description**

Computes an ECMLE estimate of the log marginal likelihood from posterior draws, corresponding log unnormalized posterior evaluations, and a function that evaluates the log unnormalized posterior density at arbitrary points.

**Usage**

```
ecmle(
  post_samples,
  lps,
  log_post_fn,
  hpd_level = 0.75,
  subsample_frac = 0.05,
  r_max = NULL,
  bisect_tol = 1e-04,
  seed = NULL,
  verbose = FALSE
)
```

**Arguments**

<code>post_samples</code>	Numeric matrix of posterior draws; each row is one draw.
<code>lps</code>	Numeric vector of log unnormalized posterior values (log prior + log likelihood, up to an additive constant) evaluated at the rows of <code>post_samples</code> . Must be consistent with <code>log_post_fn</code> ; see the note below.
<code>log_post_fn</code>	A function that accepts a numeric vector of length $d$ and returns a single finite numeric scalar equal to the log unnormalized posterior density at that point. Called repeatedly during ellipsoid construction, so it should be reasonably fast. Must be consistent with <code>lps</code> ; see the note below.
<code>hpd_level</code>	Fraction in $(0, 1)$ used to define the HPD region. Default <code>0.75</code> .
<code>subsample_frac</code>	Fraction of HPD points retained for candidate ellipsoid centers. Default <code>0.05</code> .
<code>r_max</code>	Upper bracketing radius used by the directional bisection solver. If <code>NULL</code> , estimated from the HPD points.
<code>bisect_tol</code>	Tolerance for the directional radius solver. Default <code>1e-4</code> .
<code>seed</code>	Optional integer seed used for HPD subsampling.
<code>verbose</code>	Logical; if <code>TRUE</code> , emits basic progress information.

**Details**

The algorithm splits posterior draws into two halves. The first half is used to define and pack ellipsoids inside a high-posterior-density region. The second half is used to estimate the coverage-adjusted harmonic-mean quantity that yields the final estimate.

**Value**

An object of class "ecmle", a list with components:

log\_marginal\_likelihood Final log marginal likelihood estimate.

log\_marginal\_likelihood\_iter Running estimate over the second half-sample.

ellipsoids List of fitted ellipsoids.

total\_volume Total ellipsoid volume.

n\_ellipsoids Number of ellipsoids.

points\_in\_ellipsoids Number of evaluation points inside the ellipsoids.

n\_samples Number of evaluation points in the second half-sample.

coverage\_rate Fraction of evaluation points covered by the ellipsoids.

hpd\_level HPD level used in the fit.

**Note**

**Consistency of lps and log\_post\_fn is critical.** Both must evaluate the same log unnormalized posterior (log prior + log likelihood) up to the same additive constant. If they differ by more than a constant the marginal likelihood estimate will be incorrect.

**Examples**

```
set.seed(1)
d <- 2
b <- rep(-1, d - 1)
a <- rep(0, d - 1)

Y_bar <- rosenbrock_generate_data(
  theta_true = seq(1, 2.8, length.out = d),
  n = 20, b = b, a = a, sigma = 8
)
samps <- rosenbrock_exact_posterior(Y_bar, n = 20,
  b = b, a = a, sigma = 8, n_samples = 500L)
lps <- rosenbrock_log_post_vec(samps, Y_bar,
  n = 20, b = b, a = a, sigma = 8)
log_post_fn <- function(theta)
  rosenbrock_log_post(theta, Y_bar, n = 20, b = b, a = a, sigma = 8)

fit <- ecmle(samps, lps, log_post_fn, hpd_level = 0.75, seed = 1L)
fit
summary(fit)
plot_ecmle_2d(fit, post_samples = samps)
```

**Description**

S3 methods for inspecting ECMLE fits.

**Usage**

```
## S3 method for class 'ecmle'  
print(x, ...)  
  
## S3 method for class 'ecmle'  
summary(object, ...)  
  
## S3 method for class 'summary.ecmle'  
print(x, ...)  
  
## S3 method for class 'ecmle'  
plot(  
  x,  
  y,  
  xlab = "Iteration",  
  ylab = "Running log marginal likelihood",  
  main = "ECMLE running estimate",  
  ...  
)
```

**Arguments**

x	An object of class "ecmle" or "summary.ecmle".
object	An object of class "ecmle".
y	Unused.
xlab	X-axis label for the plot.
ylab	Y-axis label for the plot.
main	Plot title.
...	Unused or additional graphical parameters.

**Value**

summary.ecmle() returns an object of class "summary.ecmle". The print and plot methods return their input invisibly.

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pair_plot	<i>Pair plot of posterior samples</i>
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### Description

Produces a lower-triangular scatter/density pair plot of a posterior sample matrix. Diagonal panels display scaled marginal histograms. Lower off-diagonal panels display a 2-D pixel-density image using a blue colour ramp. Upper panels are left blank.

### Usage

```
pair_plot(samples, pixs = 1, labels = NULL)
```

### Arguments

samples	Numeric matrix with one row per posterior draw and one column per parameter. Column names are used as axis labels when labels is NULL.
pixs	Positive scalar controlling the pixel size of the 2-D density image. Smaller values produce finer resolution at higher computational cost. Default 1.
labels	Optional character vector of length ncol(samples). When NULL (default), column names of samples are used; if there are no column names, labels default to theta1, theta2, etc.

### Value

Invisibly returns NULL. Called for its side-effect (drawing a plot).

### Examples

```
set.seed(1)
d <- 2
b <- rep(-1, d - 1)
a <- rep(0, d - 1)
Y_bar <- rosenbrock_generate_data(seq(1, 2.8, length.out = d),
                                n = 20, b = b, a = a, sigma = 8)
samps <- rosenbrock_exact_posterior(Y_bar, n = 20, b = b, a = a,
                                   sigma = 8, n_samples = 2000L)
pair_plot(samps, pixs = 0.5)
```

## Description

plot\_ecmle\_2d visualises the output of `ecmle()` for a two-dimensional posterior. The second-half evaluation samples are colour-coded by ellipsoid membership (navy = inside, grey = outside) and the fitted ellipsoid boundaries are overlaid in red. For  $d > 2$  the function emits an informative message and returns `fit` invisibly without plotting.

`draw_ellipse_2d` is the low-level helper that traces a single ellipse boundary on the current graphics device. It can also be called directly to add ellipses to an existing plot.

## Usage

```
plot_ecmle_2d(
  fit,
  post_samples,
  col_inside = "navy",
  col_outside = "grey",
  col_ellipse = "red",
  pch = 19,
  cex = 0.3,
  lwd_ellipse = 1.5,
  npoints = 100L,
  xlab = expression(theta[1]),
  ylab = expression(theta[2]),
  main = NULL,
  legend_pos = "topright",
  ...
)
```

```
draw_ellipse_2d(center, Sigma, npoints = 100L, col = "red", lwd = 1.5, ...)
```

## Arguments

<code>fit</code>	An object of class "ecmle" returned by <code>ecmle()</code> .
<code>post_samples</code>	Numeric matrix of posterior draws (N by d); the same matrix passed to <code>ecmle()</code> . Only the second half-sample is used for the scatter.
<code>col_inside</code>	Colour for samples inside at least one ellipsoid. Default "navy".
<code>col_outside</code>	Colour for samples outside all ellipsoids. Default "grey".
<code>col_ellipse</code>	Colour for ellipsoid boundaries. Default "red".
<code>pch</code>	Point character; default 19 (solid circle).
<code>cex</code>	Point size; default 0.3.
<code>lwd_ellipse</code>	Line width for ellipse boundaries; default 1.5.

<code>npoints</code>	Integer; number of boundary points per ellipse. Default 100.
<code>xlab</code>	X-axis label; default <code>expression(theta[1])</code> .
<code>ylab</code>	Y-axis label; default <code>expression(theta[2])</code> .
<code>main</code>	Plot title. When NULL (default) a title of the form "ECMLE: k ellipsoids" is constructed automatically.
<code>legend_pos</code>	Legend position string passed to <code>legend()</code> , or NULL to suppress the legend. Default "topright".
<code>center</code>	Numeric vector of length 2; ellipse center.
<code>Sigma</code>	2 by 2 symmetric positive-definite covariance matrix defining the ellipse boundary.
<code>col</code>	Line colour for <code>draw_ellipse_2d</code> ; default "red".
<code>lwd</code>	Line width for <code>draw_ellipse_2d</code> ; default 1.5.
<code>...</code>	Additional graphical parameters passed to <code>plot()</code> or <code>lines()</code> .

### Value

`plot_ecmle_2d` returns fit invisibly. `draw_ellipse_2d` returns NULL invisibly. Both are called for their side-effect (drawing on the active graphics device).

### Examples

```
set.seed(42)
post_samps <- cbind(rnorm(400), rnorm(400))
lps <- apply(post_samps, 1, function(z) sum(dnorm(z, log = TRUE)))
log_post_fn <- function(theta) sum(dnorm(theta, log = TRUE))

fit <- ecmle(post_samps, lps, log_post_fn, hpd_level = 0.75, seed = 1L)
plot_ecmle_2d(fit, post_samples = post_samps)

# draw_ellipse_2d standalone usage
plot(0, 0, xlim = c(-3, 3), ylim = c(-3, 3), type = "n", asp = 1,
     xlab = "", ylab = "")
draw_ellipse_2d(center = c(0, 0),
               Sigma = matrix(c(1, 0.4, 0.4, 1), 2, 2))
```

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rosenbrock

*Rosenbrock (banana) posterior*

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### Description

Functions for the Rosenbrock (banana-shaped) posterior, providing data generation, exact posterior sampling, and log-posterior evaluation.

`rosenbrock_generate_data` draws one sufficient statistic vector. `rosenbrock_exact_posterior` draws exact posterior samples. `rosenbrock_log_post` evaluates the log-posterior at a single parameter vector (suitable as the `log_post_fn` argument to `ecmle()`). `rosenbrock_log_post_vec` is the vectorised version over a sample matrix.

**Usage**

```
rosenbrock_generate_data(theta_true, n, b, a, sigma)

rosenbrock_exact_posterior(Y_bar, n, b, a, sigma, n_samples = 10000L)

rosenbrock_log_post(theta, Y_bar, n, b, a, sigma)

rosenbrock_log_post_vec(theta_matrix, Y_bar, n, b, a, sigma)
```

**Arguments**

<code>theta_true</code>	Numeric vector of length $d$ ; true parameter values.
<code>n</code>	Positive integer; number of observations used to form the sufficient statistic.
<code>b</code>	Numeric vector of length $d - 1$ ; curvature (banana) parameters.
<code>a</code>	Numeric vector of length $d - 1$ ; shift parameters.
<code>sigma</code>	Positive scalar; observation standard deviation.
<code>Y_bar</code>	Numeric vector of length $d$ ; sufficient statistic, typically the output of <code>rosenbrock_generate_data()</code> .
<code>n_samples</code>	Positive integer; number of posterior draws to generate. Default 10000.
<code>theta</code>	Numeric vector of length $d$ ; a single parameter value.
<code>theta_matrix</code>	Numeric matrix with $d$ columns; one parameter vector per row.

**Value**

`rosenbrock_generate_data` Numeric vector of length  $d$ .

`rosenbrock_exact_posterior` Numeric matrix of dimensions `n_samples` by  $d$ , with column names `theta1, ..., thetad`.

`rosenbrock_log_post` A single numeric value.

`rosenbrock_log_post_vec` Numeric vector of length `nrow(theta_matrix)`.

**Examples**

```
set.seed(1)
d <- 2
b <- rep(-1, d - 1)
a <- rep(0, d - 1)

# 1. Generate one dataset
Y_bar <- rosenbrock_generate_data(
  theta_true = seq(1, 2.8, length.out = d),
  n = 20, b = b, a = a, sigma = 8
)

# 2. Draw exact posterior samples
samps <- rosenbrock_exact_posterior(Y_bar, n = 20,
  b = b, a = a, sigma = 8, n_samples = 500L)
```

```
# 3. Evaluate the log-posterior
lps <- rosenbrock_log_post_vec(samps, Y_bar,
  n = 20, b = b, a = a, sigma = 8)

# 4. Build a log_post_fn closure for ecmlc()
log_post_fn <- function(theta)
  rosenbrock_log_post(theta, Y_bar, n = 20, b = b, a = a, sigma = 8)

fit <- ecmlc(samps, lps, log_post_fn, hpd_level = 0.75, seed = 1L)
fit
```

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