# Package 'utilityFunctionTools'

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Type Package

Title P-Spline Regression for Utility Functions and Derived Measures

**Description** Predicts a smooth and continuous (individual) utility function from utility points, and computes measures of intensity for risk and higher order risk measures (or any other measure computed with user-written function) based on this utility function and its derivatives according to the method introduced in Schneider (2017) <http://hdl.handle.net/21.11130/00-1735-0000-002E-E306-0>.

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**Repository** CRAN

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bbase Constructs a B-spline basis of degree 'deg' (Code by Paul Eilers, Package JOPS, http://statweb.lsu.edu/faculty/marx/JOPS\_0.1.0.tar.gz).

#### Description

Constructs a B-spline basis of degree 'deg' (Code by Paul Eilers, Package JOPS, http://statweb.lsu.edu/faculty/marx/JOPS\_0.1

#### Usage

bbase(x, xl = min(x), xr = max(x), ndx = 20, deg = 6)

#### Arguments

| Х   | values for the x axis.   |
|-----|--|
| xl  | minimum value, default is the minimum value of the x-values.     |
| xr  | maximum value, default is maximum value of the x-values.         |
| ndx | number of intervals to partition the distance between xl and xr. |
| deg | degree of the B-spline basis.                                    |

#### Value

a B-spline basis of degree deg and ndx + 1 internal knots.

#### Examples

```
x_finegrid <- seq(0.001, 1.0, (1.0 - 0.001) / 1000)
bbase(x_finegrid)</pre>
```

| compute_function | Computes a continuous and smooth utility function from the given util- |
|------------------|--|
|                  | ity points   |

#### Description

Computes a continuous and smooth utility function from the given utility points

compute\_function

#### Usage

```
compute_function(
   x,
   y,
   ids = NULL,
   mode = 1,
   penalty_order = 4,
   lambda_max = 10000,
   current_lambda = 1,
   ndx = 20,
   deg = 6,
   verbose = 0
)
```

#### Arguments

| x              | a matrix or dataframe containing the certainty equivalents (x-values of utility points) for a given participant in each use case.   |
|----------------|---|
| У              | can be a vector or a matrix representing the corresponding utility values (y-values of utility points).   |
| ids            | a list containing the IDs of the participants. If not given, a list with IDs from 1 to n_observations will be created.  |
| mode           | an integer between 0, 1, 2 representing the three possible modes: multiple imputation, optimal classification or 'weak' classification. Default is optimal classification (1).                                  |
| penalty_order  | highest dimension (i.e., derivative) to penalize. Must be lower than deg.   |
| lambda_max     | maximum lambda used for computing the optimal lambda. It is used only in multiple imputation (mode = 0) and optimal (mode = 1). The default value is 10000.   |
| current_lambda | lambda considered in the current iteration. Only used in multiple imputation (mode = 0) to create the combinations and as actual lambda value in 'weak' classification mode (mode = 2). The default value is 1. |
| ndx            | number of intervals to partition the distance between the lowest and highest x-values of the utility points.  |
| deg            | degree of the B-spline basis. Determines the degree of the function to be estimated. If $deg = 2$ , the estimated utility function will consist of quadratic functions.   |
| verbose        | shows some information while the program is running.  |

#### Value

A smooth and continuous utility function.

#### Examples

#### Description

Computes a continuous and smooth function according to the given utility points

#### Usage

```
compute_higher_order_risk_preferences(
    x,
    y,
    ids = NULL,
    mode = 1,
    penalty_orders = c(4),
    ndx = 20,
    deg = 6,
    measures = c("risk-arrow-pratt", "crainich-eeckhoudt", "denuit-eeckhoudt"),
    ...,
    root_filename = NULL,
    verbose = 0
)
```

| Х    | a matrix or dataframe containing the certainty equivalents (x-values of utility points) for a given participant in each use case.  |
|------|--|
| У    | can be a vector or a matrix representing the corresponding utility values (y-values of utility points).  |
| ids  | a list containing the IDs of the participants. If not given, a list with IDs from 1 to n_observations will be created.   |
| mode | an integer between 0, 1, 2 representing the three possible modes: multiple imputation, optimal classification or 'weak' classification. Default is optimal classification (1). |

#### Value

A smooth and continuous function.

#### Examples

| compute_ | measures |
|----------|----------|
|          |          |

Given a set of smooth and continuous functions, computes predefined and user-defined measures.

#### Description

Given a set of smooth and continuous functions, computes predefined and user-defined measures.

#### Usage

```
compute_measures(
  x_grids,
  coeffs,
  ids = NULL,
  ndx = 20,
  deg = 6,
  measures = c("risk-arrow-pratt", "crainich-eeckhoudt", "denuit-eeckhoudt"),
  ...
)
```

#### Arguments

| x_grids  | a dataframe of vectors of x values for a smooth and continuous function.  |
|----------|---|
| coeffs   | a dataframe of coefficients for a smooth and continous function for each partic-<br>ipant.  |
| ids      | a list containing the IDs of the participants. If not given, a list with IDs from 1 to n_observations will be created.  |
| ndx      | number of intervals to partition the distance between the lowest and highest x-values of the utility points.  |
| deg      | degree of the B-spline basis. Determines the degree of the function to be estimated. If $deg = 2$ , the estimated utility function will consist of quadratic functions. |
| measures | a vector of measures to be computed.  |
|          | additional parameters for user-defined measures.  |

#### Value

A set of measurements.

#### Examples

```
x <- rbind(seq(0.000002, 1.0, (1.0 - 0.000002) / 1000),
           seq(0.001, 1.0, (1.0 - 0.001) / 1000),
           seq(0.0004, 1.0, (1.0 - 0.0004) / 1000))
y <- rbind(seq(0.000002, 1.0, (1.0 - 0.000002) / 15),
           seq(0.001, 1.0, (1.0 - 0.001) / 15),
           seq(0.0004, 1.0, (1.0 - 0.0004) / 15))
compute_measures(x, y, ndx = 10, deg = 6)
# x_finegrid, coeff, ndx, deg are always there to be used
# The function should have additional unknown arguments (...) if the given parameters are not used
risk_arrow_pratt <- function(x_finegrid, coeff, ndx, deg){</pre>
  dy_rd <- derivative(x_finegrid, coeff, 1, ndx, deg)</pre>
  ddy_rd <- derivative(x_finegrid, coeff, 2, ndx, deg)</pre>
  return (-mean(ddy_rd, na.rm = TRUE) / mean(dy_rd, na.rm = TRUE))
}
measures = c("crainich-eeckhoudt", "denuit-eeckhoudt", risk_arrow_pratt)
compute_measures(x, y, ndx = 10, deg = 6, measures=measures)
```

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derivative

#### Description

Computes the derivative of a function

#### Usage

```
derivative(x, coeffs, degree = 1, ndx = 20, deg = 6)
```

#### Arguments

| х      | the x values for which the derivative should be computed.   |
|--------|---|
| coeffs | the coefficient.  |
| degree | the degree of the derivative.   |
| ndx    | number of intervals to partition the distance between the lowest and highest x-values of the utility points.  |
| deg    | degree of the B-spline basis. Determines the degree of the function to be estimated. If $deg = 2$ , the estimated utility function will consist of quadratic functions. |

#### Value

the derivative of the specified degree.

#### Examples

```
coeffs <- seq(0.000002, 1.0, (1.0 - 0.000002) / 25)
x <- seq(0.01, 1.0, (1.0 - 0.01) / 5)
derivative(x, coeffs)</pre>
```

estimate\_model Estimates the model

#### Description

Estimates the model

#### Usage

```
estimate_model(
   xi,
   yi,
   lambda = 1,
   n_penalty_dimensions = 1,
   penalty_order = 4,
   ndx = 20,
   deg = 6,
   cross_validation_mode = 0,
   return_estimate = 0,
   left_out_xi = c(),
   left_out_yi = c()
)
```

#### Arguments

| xi                    | a vector containing the certainty equivalents (x-values of utility points) for a given participant in each use case.   |  |
|-----------------------|--|--|
| yi                    | can be a vector or a matrix representing the corresponding utility values (y-values of utility points).  |  |
| lambda                | lambda is the penalization weight used to compute the initial estimate. The default value is 1.  |  |
| n_penalty_dimen       |  |  |
|                       | number of dimensions (i.e., derivatives) to penalize. Possible values are 1 or 2. The default value is 1.  |  |
| penalty_order         | highest dimension (i.e., derivative) to penalize. Must be lower than deg.  |  |
| ndx                   | number of intervals to partition the distance between the lowest and highest x-values of the utility points.   |  |
| deg                   | degree of the B-spline basis. Determines the degree of the function to be estimated. If $deg = 2$ , the estimated utility function will consist of quadratic functions.  |  |
| cross_validation_mode |  |  |
|                       | determines which cross validation mode should be used. If 0, then the cross validation method is leave-one-third-out. If 1, then the cross validation method is a theoretical leave-one-out, i.e., based on a formula. The default value is 1. |  |
| return_estimate       |  |  |
|                       | parameter that indicates whether or not to return the (initially) estimated coefficients. Default is false.  |  |
| left_out_xi           | needed for cross validation: the x-values of the points that are left out for fitting the model, so that they can be predicted   |  |
| left_out_yi           | needed for cross validation: the y-values of the points that are left out for fitting the model, so that they can be predicted   |  |

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#### Value

Returns the sum of residuals of the prediction of the left-out points using cross validation. If specified, additionally returns the estimated coefficients of the utility function (in the B-spline basis).

#### Examples

```
x <- c(0.0000000, 0.2819824, 0.3007812, 0.4375000, 0.5231934, 0.7784882, 0.8945312, 1.0000000)
y <- c(0.0000, 0.1250, 0.2500, 0.5000, 0.6250, 0.6875, 0.7500, 1.0000)
estimate_model(x, y, .5)
```

evaluate\_cross\_validation

Evaluates the cross validation function.

#### Description

Evaluates the cross validation function.

#### Usage

```
evaluate_cross_validation(
   xi,
   yi,
   lambda = 1,
   n_penalty_dimensions = 1,
   penalty_order = 4,
   ndx = 20,
   deg = 6,
   cross_validation_mode = 0
)
```

| xi                   | a vector containing the certainty equivalents (x-values of utility points) for a given participant in each use case. |  |
|----------------------|--|--|
| уі                   | can be a vector or a matrix representing the corresponding utility values (y-values of utility points).              |  |
| lambda               | lambda is the penalization weight used to compute the initial estimate. The default value is 1.                      |  |
| n_penalty_dimensions |  |  |
|                      | number of dimensions (i.e., derivatives) to penalize. Possible values are 1 or 2. The default value is 1.            |  |
| penalty_order        | highest dimension (i.e., derivative) to penalize. Must be lower than deg.  |  |
| ndx                  | number of intervals to partition the distance between the lowest and highest x-values of the utility points.         |  |

deg degree of the B-spline basis. Determines the degree of the function to be estimated. If deg = 2, the estimated utility function will consist of quadratic functions.

cross\_validation\_mode

determines which cross validation mode should be used. If 0, then the cross validation method is leave-one-third-out. If 1, then the cross validation method is a theoretical leave-one-out, i.e., based on a formula. The default value is 1.

#### Value

Returns, for the given utility points and (possibly default) settings, the predictive quality of the estimated utility function according to cross validation as a function of a specified penalty weight lambda.

#### Examples

```
x <- c(0.0000000, 0.2819824, 0.3007812, 0.4375000, 0.5231934, 0.7784882, 0.8945312, 1.0000000)
y <- c(0.0000, 0.1250, 0.2500, 0.5000, 0.6250, 0.6875, 0.7500, 1.0000)
evaluate_cross_validation(x, y, .5)
```

find\_optimal\_lambda Finds an optimal penalty weight lambda given the parameters

#### Description

Finds an optimal penalty weight lambda given the parameters

#### Usage

```
find_optimal_lambda(
    xi,
    yi,
    lambda_max = 10000,
    n_penalty_dimensions = 1,
    penalty_order = 4,
    ndx = 20,
    deg = 6,
    cross_validation_mode = 0,
    grid_dim = 5
```

```
)
```

|   | a vector containing the certainty equivalents (x-values of utility points) for a given participant in each use case. |
|---|--|
| 5 | can be a vector or a matrix representing the corresponding utility values (y-values of utility points).              |

#### tpower

| lambda_max            | maximum lambda used for computing the optimal lambda. The default value is   |  |
|-----------------------|--|--|
|                       | 10000.   |  |
| n_penalty_dimensions  |  |  |
|                       | number of dimensions (i.e., derivatives) to penalize. Possible values are 1 or 2. The default value is 1.  |  |
| penalty_order         | highest dimension (i.e., derivative) to penalize. Must be lower than deg.  |  |
| ndx                   | number of intervals to partition the distance between the lowest and highest x-values of the utility points.   |  |
| deg                   | degree of the B-spline basis. Determines the degree of the function to be estimated. If $deg = 2$ , the estimated utility function will consist of quadratic functions.  |  |
| cross_validation_mode |  |  |
|                       | determines which cross validation mode should be used. If 0, then the cross validation method is leave-one-third-out. If 1, then the cross validation method is a theoretical leave-one-out, i.e., based on a formula. The default value is 1. |  |
| grid_dim              | dimension of the search grid for the initial grid search before the actual opti-<br>mization. Default value is 5.  |  |

#### Value

the optimal lambda for the given set of utility points and (possibly default) settings according to the specified cross validation method.

#### Examples

x <- c(0.0000000, 0.2819824, 0.3007812, 0.4375000, 0.5231934, 0.7784882, 0.8945312, 1.0000000) y <- c(0.0000, 0.1250, 0.2500, 0.5000, 0.6250, 0.6875, 0.7500, 1.0000) find\_optimal\_lambda(x, y)

| tpower | Truncated p-th power function. Helper function for creat-  |
|--------|--|
|        | ing the B-Spline basis (Code by Paul Eilers, Package JOPS, |
|        | http://statweb.lsu.edu/faculty/marx/JOPS_0.1.0.tar.gz)     |

#### Description

Truncated p-th power function. Helper function for creating the B-Spline basis (Code by Paul Eilers, Package JOPS, http://statweb.lsu.edu/faculty/marx/JOPS\_0.1.0.tar.gz)

#### Usage

tpower(x, t, p)

| х | Function value.                              |
|---|--|
| t | Point of truncation.                         |
| р | degree of the truncated polynomial function. |

#### tpower

## Value

Returns a piece-wise defined basis functions for x > t.

### Examples

tpower(1, 2, 3)

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