

Package ‘Rfmtool’

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Description Various tools for handling fuzzy measures, calculating Shapley value and interaction index, Choquet and Sugeno integrals, as well as fitting fuzzy measures to empirical data are provided. Construction of fuzzy measures from empirical data is done by solving a linear programming problem by using 'lpsolve' package, whose source in C adapted to the R environment is included. The description of the basic theory of fuzzy measures is in the manual in the Doc folder in this package. Please refer to the following:

- [1] <<https://personal-sites.deakin.edu.au/~gleb/fmtools.html>>
- [2] G. Beliakov, H. Bustince, T. Calvo, 'A Practical Guide to Averaging', Springer, (2016, ISBN: 978-3-319-24753-3).
- [3] G. Beliakov, S. James, J-Z. Wu, 'Discrete Fuzzy Measures', Springer, (2020, ISBN: 978-3-030-15305-2).

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`fm`*Rfmtool package*

Description

This function shows a list of function included in this toolbox

Usage

`fm()`

Details

The following functions involve the parameters v (the array containing the fuzzy measure in standard representation) or Mob (in Möbius representation), n - the dimension and m = 2^n. The values of the fuzzy measure always obey the binary ordering.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

`fm()`

`fm.add_pair_sparse`*Function for adding a pair to the sparse fuzzy measure*

Description

This is used for populating capacities which Add a pair v_ij to the structure, their Indices are 1-based.

Usage

`fm.add_pair_sparse(i, j, v, envsp = NULL)`

Arguments

i	One of the indices which are 1-based
j	One of the indices which are 1-based
v	The value to be added.
envsp	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n)

Value

output The output is an added pair v_ij to the structure.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
tups<-vector()
tupsidx<-vector()
envsp <- fm.PrepareSparseFM(n, tups,tupsidx)
envsp <-fm.add_pair_sparse(1,2, 0.4, envsp)
envsp <-fm.add_pair_sparse(1,3, 0.3, envsp)
envsp
envsp <- fm.FreeSparseFM(envsp)
```

fm.add_singletons_sparse

Function for adding singletons to the sparse fuzzy measure

Description

This is used for adding singletons to the structure.

Usage

```
fm.add_singletons_sparse(v, envsp=NULL)
```

Arguments

v The vector of singletons of size n.

envsp Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

output The output is added singletons to the structure.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
tups<-vector()
tupsidx<-vector()
envsp <- fm.PrepareSparseFM(n, tups,tupsidx)

envsp <- fm.add_singletons_sparse(c(0, 0.3, 0.5),envsp)
```

fm.add_tuple_sparse *Function for adding singletons to the sparse fuzzy measure*

Description

This is used for populating capacities which Add a tuple of size tupsize to the structure whose Indices are 1-based in tuple.

For populating capacities, adds a whose 1-based indices are in tuple

Usage

```
fm.add_tuple_sparse( tuple, v, envsp=NULL)
```

Arguments

tuple	Collection of objects. It is a list of cardinalities of the nonzero tuples (cardinality, tuple composition)
v	The value of the tuple to be added
envsp	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

output	The output is adding a tuple of size tupsize
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 4
tups<-vector()
tupsidx<-vector()
envsp <- fm.PrepareSparseFM(n, tups,tupsidx)
envsp <- fm.add_tuple_sparse(c(1,2,3),0.2,envsp)
envsp <- fm.add_tuple_sparse(c(1,3,4),0.3,envsp)
```

fm.Banzhaf*Banzhaf value computation function***Description**

Calculates the Banzhaf indices of input criteria from general fuzzy measure.

Usage

```
fm.Banzhaf(v, env=NULL)
```

Arguments

- | | |
|-----|--|
| v | Fuzzy measure in general representation. |
| env | Environment variable obtained from fm.Init(n). |

Value

- | | |
|--------|--|
| output | The output is an array of size n, which contain Banzhaf indices of input criteria. |
|--------|--|

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.Banzhaf(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1), env)

env<-fm.Free(env)
```

fm.Banzhaf2addMob*Function for calculating Banzhaf values of 2-additive fuzzy measure in Mobius representation***Description**

Calculate the Banzhaf values of a 2-additive fuzzy measure for n inputs given in Mobius representation. The results are in arrays.

Usage

```
fm.Banzhaf2addMob(n, Mob)
```

Arguments

n	Number of inputs
Mob	Fuzzy measure value in Mobius representation

Value

output The output is an array of size n, which contain Banzhaf indices of input criteria.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
Banzhaf <- fm.Banzhaf2addMob(3, c(0.2, 0.3, 0.5, -0.2, 0.4, 0.1))
```

fm.BanzhafMob

Banzhaf value computation function in Mobius representation

Description

Calculates the Banzhaf indices of input criteria from general fuzzy measure in Mobius representation.

Usage

```
fm.BanzhafMob(Mob, env=NULL)
```

Arguments

Mob	Fuzzy measure in Mobius representation.
env	Environment variable obtained from fm.Init(n).

Value

output The output is an array of size n, which contain Banzhaf indices of input criteria.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.BanzhafMob(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1),env)
env<-fm.Free(env)
```

fm.BanzhafMob_sparse *Banzhaf values computation function in sparse representation*

Description

Calculates Banzhaf values vectors of size n of a sparse fuzzy measure

Usage

```
fm.BanzhafMob_sparse(n, envsp=NULL)
```

Arguments

n	The size of values vectors
envsp	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

output	The output is Banzhaf values vectors of size n of a sparse fuzzy measure.
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```

n <- 3
tups<-vector()
tupsidx<-vector()
envsp <- fm.PrepareSparseFM(n, tups,tupsidx)
envsp <- fm.add_singletons_sparse(c(0.2,0.1,0.2),envsp)
envsp <- fm.add_pair_sparse(1,2,0.4,envsp);

fm.BanzhafMob_sparse(3, envsp)

envsp <- fm.FreeSparseFM(envsp)

```

`fm.Bipartition` *Bipartition interaction index computation function*

Description

Calculates the Bipartition interaction indices of input criteria from general fuzzy measure.

Usage

```
fm.Bipartition(v,env=NULL)
```

Arguments

- | | |
|------------------|--|
| <code>v</code> | Fuzzy measure in general representation. |
| <code>env</code> | Environment variable obtained from fm.Init(n). |

Value

- | | |
|---------------------|--|
| <code>output</code> | The output is an array of size 2^n , which contain bipartition interaction indices of input criteria coalitions. |
|---------------------|--|

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.Bipartition(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1),env)
env<-fm.Free(env)
```

`fm.BipartitionBanzhaf` *Bipartition Banzhaf interaction index computation function*

Description

Calculates the Banzhaf Bipartition interaction indices of input criteria from general fuzzy measure.

Usage

```
fm.BipartitionBanzhaf(v,env=NULL)
```

Arguments

- | | |
|------------------|--|
| <code>v</code> | Fuzzy measure in general representation. |
| <code>env</code> | Environment variable obtained from fm.Init(n). |

Value

- output** The output is an array of size 2^n , which contain Banzhaf bipartition interaction indices of input criteria coalitions.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.BipartitionBanzhaf(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1),env)
env<-fm.Free(env)
```

fm.check_convexity_monotonicity_mob

Function for checking supermodularity of the set function v in Mobius representation

Description

Checks supermodularity of the set function v in Mobius representation using standard check.

Usage

```
fm.check_convexity_monotonicity_mob(v, len, env=NULL)
```

Arguments

- v** matrix v stores fuzzy measurements consecutively in cardinal order v.
len this is the length of array Mob (this array is usually smaller than 2^n), and is computed by fm.fm_arraysize_kadd(N, Kadd).
env Environment variable obtained from fm.Init(n).

Value

- output** The output is 1 or 0 to check for monotonicity.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
step <- 0.001
Fn <- NULL
Option<- 3
fuzzymeasures <- fm.generate_fm_randomwalk(1, 3, 2, 1000, Option, step, Fn, env)
len <- fuzzymeasures$length
check <- fm.check_convexity_monotonicity_mob(fuzzymeasures$V, len, env)
```

fm.check_monotonicity *Function for checking monotonicity of the set function v*

Description

Checks monotonicity of the set function v in standard representation using insert sort.

Usage

```
fm.check_monotonicity(v, env=NULL)
```

Arguments

v	matrix v stores fuzzy measurements consecutively in cardinal order.
env	Environment variable obtained from fm.Init(n).

Value

output	The output is 1 or 0 to check for monotonicity.
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
v <- fm.generate_fm_sorting(1, 1000, 0, env)
monotonicity <- fm.check_monotonicity(v, env)
```

fm.check_monotonicity_mob

Function for checking monotonicity of the set function v in Mobius representation.

Description

Checks monotonicity of the set function v in Mobius representation using standard check.

Usage

```
fm.check_monotonicity_mob(v, len, env=NULL)
```

Arguments

v	matrix v stores fuzzy measurements consecutively in cardinal order.
len	this is the length of array Mob (this array is usually smaller than 2^n), and is computed by fm_arraysize_kadd(N, Kadd)
env	Environment variable obtained from fm.Init(n).

Value

output	The output is 1 or 0 to check for monotonicity
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
step <- 100
Fn <- NULL
Option<- 3
fuzzymeasures <- fm.generate_fm_randomwalk(1, 3, 2, 1000, Option, step, Fn, env)
len <- fuzzymeasures$length
check <- fm.check_monotonicity_mob(fuzzymeasures$V, len, env)
check
```

`fm.check_monotonicity_mob_2additive`

Function for checking the monotonicity of the 2-additive set function v in Mobius representation.

Description

Check the monotonicity of the 2-additive set function v in Mobius representation using fast check.

Usage

```
fm.check_monotonicity_mob_2additive(v, n, temp=NULL)
```

Arguments

v	Random 2-additive fuzzy measure in Mobius representation.
n	Number of inputs
temp	Auxiliary array of length n^2 (e.g: array(0.0,n*n)). It may or may not be specified (if speed matters, then preallocate it).

Value

output	The output is 1 or 0 to check for monotonicity.
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
v <- fm.generate_fm_2additive(1, 10)
n <- 10
v$len
v$V
check <- fm.check_monotonicity_mob_2additive(v$V, n)
check

temp <- array(0.0,10*10);
check <- fm.check_monotonicity_mob_2additive(v$V, n, temp)
check
```

fm.check_monotonicity_sort_insert*Function for checking monotonicity of the set function v***Description**

Checks monotonicity of the set function v in standard representation using insert sort.

Usage

```
fm.check_monotonicity_sort_insert(v, indices, env=NULL)
```

Arguments

v	matrix v stores fuzzy measurements consecutively in cardinal order.
indices	The indices can be used at subsequent steps of monotonicity verification. This function is called after merge sort, so the indices are already precomputed.
env	Environment variable obtained from fm.Init(n).

Value

output	The output is a list of components (True/False, indices, values). The indices and values can be used at subsequent steps of monotonicity verification (e.g., values slightly perturbed)
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
v <- fm.generate_fm_sorting(1, 1000, 0, env)
out <- fm.check_monotonicity_sort_merge(v, NULL, env)
out$V[1] = out$V[1] *1.1
out<- fm.check_monotonicity_sort_insert(out$V, out$index, env)
out$out
```

fm.check_monotonicity_sort_merge*Function for checking monotonicity of the set function v*

Description

Checks monotonicity of the set function v in standard representation using merge sort.

Usage

```
fm.check_monotonicity_sort_merge(v, indices=NULL, env=NULL)
```

Arguments

v	matrix v stores fuzzy measurements consecutively in cardinal order.
indices	The indices can be used at subsequent steps of monotonicity verification. Initially indices need not be specified
env	Environment variable obtained from fm.Init(n).

Value

output	The output is a list of components (True/False, indices, values). The indices and values can be used at subsequent steps of monotonicity verification (e.g., values slightly perturbed)
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
v <- fm.generate_fm_sorting(1, 1000, 0, env)
v
measure <- fm.check_monotonicity_sort_merge(v,NULL, env)

print(measure$out)

measure$V[1] = measure$V[1] *1.1
measure <- fm.check_monotonicity_sort_merge(measure$V, measure$index, env)
```

fm.Choquet*Choquet integral computation function***Description**

Calculates the value of a discrete Choquet integral of input x , with fuzzy measure in general representation.

Usage

```
fm.Choquet(x, v, env=NULL)
```

Arguments

- | | |
|------------------|--|
| <code>x</code> | Input vector of size n , containing utility value of input criteria. x is in [0,1]. |
| <code>v</code> | The general fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data. |
| <code>env</code> | Environment variable obtained from <code>fm.Init(n)</code> . |

Value

- | | |
|---------------------|--|
| <code>output</code> | The output is a single value of the computed Choquet integral. |
|---------------------|--|

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.Choquet(c(0.6, 0.3, 0.8), c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1),env)
env<-fm.Free(env)
```

fm.Choquet2addMob*Function for calculating Choquet integral value for 2-additive fuzzy measure in Mobius representation***Description**

Calculates the Choquet integral value of a 2-additive fuzzy measure for n inputs given in Mobius representation.

Usage

```
fm.Choquet2addMob(n, x, Mob)
```

Arguments

n	Number of inputs
x	Input vector of size n, containing utility value of input criteria. x is in [0,1].
Mob	The Mobius fuzzy measure of size m=2^n. Its values can be provided by users, or by estimating from empirical data.

Value

output	The output is the Choquet integral value in Mobius representation.
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
Choquet <- fm.Choquet2addMob(3, c(0.2,0.5,0.4), c(0.2, 0.3, 0.5, -0.2, 0.4, 0.1))
```

`fm.ChoquetCoMobKInter` function for calculating Choquet integral value with respect to dual k-interactive fuzzy measure in Mobius representation

Description

Calculates the Choquet integral of x with respect to dual k-interactive fuzzy measure in Mobius representation.

Usage

```
fm.ChoquetCoMobKInter(x, Mob, kadd, env=NULL)
```

Arguments

x	Input vector of size n, containing utility value of input criteria. x is in [0,1].
Mob	The Mobius fuzzy measure of size m=2^n. Its values can be provided by users, or by estimating from empirical data
kadd	is the value of k-additivity, which is used for reducing the complexity of fuzzy measures. kadd is defined as an optional argument, its default value is kadd = n. kadd is k in k-additive f-measure, 1 < kadd < n+1; if kdd=n - f.m. is unrestricted.
env	Environment variable obtained from fm.Init(n).

Value

output	The output is the Choquet integral value in Mobius representation.
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env <-fm.Init(4)
step <- 0.0001
Fn <- NULL
fuzzymeasures <- fm.generate_fm_kinteractivivedualconvex(1, 4, 2, 1000, step, Fn, env)
fuzzymeasures
env

fm.ChoquetCoMobKInter(c(0.2,0.5,0.4,0.1), fuzzymeasures$V, 2, env)
env<-fm.Free(env)
```

fm.ChoquetKinter

Choquet integral value computation function in standard representation wrt k-interactive fuzzy measure

Description

This is an alternative calculation of the Choquet integral from the fuzzy measure in Mobius representation.

Usage

```
fm.ChoquetKinter(x, v, kint, env)
```

Arguments

- | | |
|------|---|
| x | Input vector of size n, containing utility value of input criteria. x is in [0,1]. |
| v | The fuzzy measure of size less than m=2^n. Its values can be provided by users, or by estimating from empirical data. |
| kint | the k-interactive parameter, must be smaller than n. |
| env | Environment variable obtained from fm.Init(n). |

Value

- | | |
|--------|--|
| output | The output is a single value of the computed Choquet integral. |
|--------|--|

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.ChoquetKinter(c(0.6,0.3,0.8),c(0,0.3,0.5,0.6,0.4,0.8,0.7,1),2,env)
env<-fm.Free(env)
```

fm.ChoquetMob

Choquet integral value computation function in Mobius representation

Description

This is an alternative calculation of the Choquet integral from the fuzzy measure in Mobius representation.

Usage

```
fm.ChoquetMob(x, Mob, env=NULL)
```

Arguments

- | | |
|-----|--|
| x | Input vector of size n, containing utility value of input criteria. x is in [0,1]. |
| Mob | The Mobius fuzzy measure of size m=2^n. Its values can be provided by users, or by estimating from empirical data. |
| env | Environment variable obtained from fm.Init(n). |

Value

- | | |
|--------|---|
| output | The ouput is a single value of the computed Choquet integral. |
|--------|---|

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.ChoquetMob(c(0.2,0.5,0.4), c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1),env)
```

fm.ChoquetMob_sparse Choquet integral computation function in sparse representation

Description

Calculates the Choquet integral in Mobius sparse representation.

Usage

```
fm.ChoquetMob_sparse(x, envsp=NULL)
```

Arguments

- | | |
|-------|--|
| x | Input vector of size n, containing utility value of input criteria. x is in [0,1]. |
| envsp | Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n). |

Value

- | | |
|--------|---|
| output | The output is the Choquet integral in Mobius sparse representation. |
|--------|---|

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
envsp <- fm.PrepareSparseFM(n, vector(), vector())
envsp <- fm.add_singletons_sparse(c(0.2,0.1,0.2),envsp)
envsp <- fm.add_pair_sparse(1,2,0.4,envsp);

ChoquetMobsparse <- fm.ChoquetMob_sparse(c(0.1,0.05,0.2),envsp)
ChoquetMobsparse
envsp <- fm.FreeSparseFM(envsp)
```

fm.ConstructLambdaMeasure

Function for Constructing Lambda

Description

Finds the value of lambda and calculates the rest of the values of the fuzzy measure, given its values at singletons; singletons is an array of size n. The outputs are lambda and v, v is in standard representation and binary ordering.

Usage

```
fm.ConstructLambdaMeasure(singletons, env)
```

Arguments

singletons	Singletons is an array of n.
env	Environment variable obtained from fm.Init(n).

Value

output	The ouput is the list (lambda, measure), where measure is a fuzzy measure in standard representation.
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
w <- fm.ConstructLambdaMeasure(c(0, 0.3, 0.5), env)
```

fm.ConstructLambdaMeasureMob

Function for Constructing Lambda in Mobius representation

Description

Finds the value of lambda and calculates the rest of the values of the fuzzy measure, given its values at singletons; singletons is an array of size n. The outputs are lambda and measure, measure is in Mobius representation.

Usage

```
fm.ConstructLambdaMeasureMob(singletons, env)
```

Arguments

singletons	Singletons is an array of n.
env	Environment variable obtained from fm.Init(n).

Value

output	The ouput is the list (lambda, measure), where measure is a fuzzy measure in Mobius representation.
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
w <- fm.ConstructLambdaMeasureMob(c(0, 0.3, 0.5), env)
w$measure
fm.Free(env)
```

fm.ConvertCoMob2Kinter

Function for dual k-interactive fuzzy measure from Mobius to standard representation

Description

Converts dual k-interactive fuzzy measure from Mobius to standard representation.

Usage

```
fm.ConvertCoMob2Kinter(Mob,kadd, fullmu, env=NULL)
```

Arguments

Mob	Mobius fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
kadd	is the value of k-additivity, which is used for reducing the complexity of fuzzy measures. kadd is defined as an optional argument, its default value is kadd = n. kadd is k in k-additive f-measure, $1 < kadd < n+1$; if kadd=n - f.m. is unrestricted.
fullmu	Integer flag. is 1 then all 2^n are allocated, otherwise a more compact representation fo rk-interactive fuzzy measures is used.
env	Environment variable obtained from fm.Init(n).

Value

output	The output is k-interactive fuzzy measure standard representation
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```

env <- fm.Init(4)
fullmu <- 0
step<-0.001
Fn <- NULL
fuzzymeasures <- fm.generate_fm_kinteractivivedualconvex(1, 4, 2, 1000,
step, Fn, env)

fm.ConvertCoMob2Kinter(fuzzymeasures$V, 2, fullmu, env )

```

fm.dualm

Function for calculating dual of fuzzy measure

Description

Calculates the dual of fuzzy measure v, returns it as value of the function (array of size m).

Usage

```
fm.dualm(v, env=NULL)
```

Arguments

- | | |
|-----|--|
| v | General fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data. |
| env | Environment variable obtained from fm.Init(n). |

Value

- | | |
|--------|--|
| output | The output is an array of size m with the dual of fuzzy measure v. |
|--------|--|

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```

env<-fm.Init(3)
w <- fm.dualm(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1),env)

```

fm.dualmMob*Dualm computation function in Mobius representation***Description**

Calculates the dual of fuzzy measure v, returns it as value of the function (array of size m).

Usage

```
fm.dualmMob(Mob, env=NULL)
```

Arguments

- | | |
|-----|---|
| Mob | Mobius fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from emperical data. |
| env | Environment variable obtained from fm.Init(n). |

Value

- | | |
|--------|---|
| output | The ouput is an array of size m with the dual of fuzzy measure. |
|--------|---|

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
w <- fm.dualmMob(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1),env)
```

fm.dualMobKadd*Function for calculating dual of k-additive fuzzy measure in Mobius representation***Description**

Calculates the dual of a k-additive fuzzy measures for n inputs.

Usage

```
fm.dualMobKadd(Mob, env = NULL, kadd = "NA")
```

Arguments

Mob	Mobius fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from fm.Init(n).
kadd	Value of k-interactivity, which is used for reducing the complexity of fuzzy measures. It is defined as an optional argument

Value

output	The output is the dual of a k-additive fuzzy measures for n inputs
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
dualMob_Kadd <- fm.dualMobKadd(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1), env,2)
```

fm.EntropyChoquet *Entropy of fuzzy measure*

Description

Calculates entropy value of the Choquet integral for the fuzzy measure v in general representation

Usage

```
fm.EntropyChoquet(v,env)
```

Arguments

v	General fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from fm.Init(n).

Value

output	The ouput is the entropy value of the Choquet integral for the fuzzy measure.
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.EntropyChoquet(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1))
```

fm.EntropyChoquetMob *Entropy Choquet computation function in Mobius representation*

Description

Calculates entropy value of the Choquet integral for the fuzzy measure v in Mobius representation

Usage

```
fm.EntropyChoquetMob(Mob, env)
```

Arguments

Mob	Mobius fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from fm.Init(n).

Value

output	The output is entropy value of the Choquet integral for the fuzzy measure.
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.EntropyChoquetMob(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1), env)
```

fm.errorcheck*Basic error check*

Description

This function checks that the environment variable is internally consistent.

Usage

```
fm.errorcheck(env)
```

Arguments

env	Environment variable obtained from fm.Init(n).
-----	--

Value

output	The output is TRUE or FALSE.
--------	------------------------------

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.errorcheck(env)
```

fm.expand_2add_full *Function for exporting full representation of 2-additive capacity*

Description

From sparse to full representation of 2-additive capacity (singletons and pairs, augmented with 0s).

Usage

```
fm.expand_2add_full(n, envsp=NULL)
```

Arguments

n	Number of inputs
---	------------------

envsp	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).
-------	--

Value

- output** The output is a sparse to full representation of 2-additive capacity (singletons and pairs, augmented with 0s)

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
envsp <- fm.PrepareSparseFM(n, vector(), vector())
envsp <- fm.add_singletons_sparse(c(0.2,0.1,0.2),envsp)
envsp <- fm.add_pair_sparse(1,2,0.4,envsp);

cap2add <- fm.expand_2add_full(n,envsp)
cap2add

envsp <- fm.FreeSparseFM(envsp)
```

fm.expand_sparse_full Function for exporting full capacity from sparse representation

Description

Exports from sparse to full capacity.

Usage

```
fm.expand_sparse_full(n, envsp=NULL)
```

Arguments

- n** Number of inputs.
envsp Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

- output** Exports from sparse to full capacity.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n<-3
envsp <- fm.PrepareSparseFM(n, vector(), vector())
envsp <- fm.add_singletons_sparse(c(0.2,0.1,0.2),envsp)
envsp <- fm.add_pair_sparse(1,2,0.4,envsp);

cap <- fm.expand_sparse_full(n, envsp)
cap

envsp <- fm.FreeSparseFM(envsp)
```

`fm.export_maximal_chains`

Function for exporting maximal chains

Description

Returns in mc the arrays of maximal chains (there are $n!$ such arrays) of a fuzzy measure v. Each maximal chain corresponds to the coefficients of a linea. function on the respective simplex

Usage

```
fm.export_maximal_chains(v, env = NULL)
```

Arguments

- | | |
|-----|--|
| v | Fuzzy measure in general representation. |
| env | Environment variable obtained from fm.Init(n). |

Value

- | | |
|--------|---|
| output | The output is mc the arrays of maximal chains |
|--------|---|

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
exportmaximalchains <- fm.export_maximal_chains(
c(0, 0.00224, 0.0649, 0.510, 0.00965, 0.374, 0.154, 1),env)
```

fm.fitting*Fuzzy Measure Fitting function***Description**

Estimate values of the fuzzy measures from empirical data.

Usage

```
fm.fitting(data, env=NULL, kadd="NA")
```

Arguments

data	Empirical data set in pairs $(x_1, y_1), (x_2, y_2), \dots, (x_d, y_d)$ where x_i in $[0,1]^n$ is a vector containing utility values of n input criteria $x_{i1}, x_{i2}, \dots, x_{in}$, y_i in $[0,1]$ is a single aggregated value given by decision makers. The data is stored as a matrix of M by $n+1$ elements, where M is the number of data instances, and n is the number of input criteria, the column $n + 1$ stores the observed aggregated value y .
env	Environment variable obtained from fm.Init(n).
kadd	The value of k-additivity, which is used for reducing the complexity of fuzzy measures. kadd is defined as an optional argument, its default value is kadd = n.

Value

output	The output is an array of size 2^n containing estimated standard fuzzy measure in binary ordering.
---------------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
d <- matrix( c( 0.00125122, 0.563568, 0.193298, 0.164338,
 0.808716, 0.584991, 0.479858, 0.544309,
 0.350281, 0.895935, 0.822815, 0.625868,
 0.746582, 0.174103, 0.858917, 0.480347,
 0.71048, 0.513519, 0.303986, 0.387631,
 0.0149841, 0.0914001, 0.364441, 0.134229,
 0.147308, 0.165894, 0.988495, 0.388044,
 0.445679, 0.11908, 0.00466919, 0.0897714,
 0.00891113, 0.377869, 0.531647, 0.258585,
 0.571167, 0.601746, 0.607147, 0.589803,
 0.166229, 0.663025, 0.450775, 0.357412,
 0.352112, 0.0570374, 0.607666, 0.270228,
 0.783295, 0.802582, 0.519867, 0.583348,
```

```

0.301941, 0.875946, 0.726654, 0.562174,
0.955872, 0.92569, 0.539337, 0.633631,
0.142334, 0.462067, 0.235321, 0.228419,
0.862213, 0.209595, 0.779633, 0.498077,
0.843628, 0.996765, 0.999664, 0.930197,
0.611481, 0.92426, 0.266205, 0.334666,
0.297272, 0.840118, 0.0237427, 0.168081),
nrow=20,
ncol=4,byrow=TRUE);
fm.fitting(d,env)

```

fm.fitting2additive *Fuzzy Measure Fitting function*

Description

Estimate values of the fuzzy measures from empirical data tailored 2-additive standard fuzzy measure.

Usage

```
fm.fitting2additive(data, options=0, indexlow, indexhigh , option1=0, orness)
```

Arguments

data	is the empirical data set in pairs $(x_{-1},y_{-1}),(x_{-2},y_{-2}),\dots,(x_{-d},y_{-d})$ where x_{-i} in $[0,1]^n$ is a vector contains utility values of n input criteria $x_{-i1},x_{-i2},\dots,x_{-in}$, y_{-i} in $[0,1]$ is a single aggregated value given by decision makers. The data is stored as a matrix of M by $n+1$ elements, where M is the number of data instances, and n is the number of input criteria, the column $n + 1$ store the observed aggregating value y .
options	options (default value is 0) 1 - lower bounds on Shapley values supplied in indexlow, 2 - upper bounds on Shapley values supplied in indexhigh, 3 - lower and upper bounds on Shapley values supplied in indexlow and indexhigh, 4 - lower bounds on all interaction indices supplied in indexlow, 5 - upper bounds on all interaction indices supplied in indexhigh, 6 - lower and upper bounds on all interaction indices supplied in indexlow and indexhigh. All these value will be treated as additional constraints in the LP.
indexlow	optional array of size n (options =1,2,3) or m (options=4,5,6) containing the lower bounds on the Shapley values or interaction indices
indexhigh	optional array of size n (options =1,2,3) or m (options=4,5,6) containing the upper bounds on the Shapley values or interaction indices
option1	if the value is 1, the interval of orness values will be fitted (and the desired low and high orness values should be provided). If 0, no additional orness constraints.
orness	optional array of size 2, for example c(0.1,1)

Value

output The output is an array containing the values of a standard fuzzy measure in binary ordering.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
d <- matrix( c( 0.00125122, 0.563568, 0.193298, 0.164338,
  0.808716, 0.584991, 0.479858, 0.544309,
  0.350281, 0.895935, 0.822815, 0.625868,
  0.746582, 0.174103, 0.858917, 0.480347,
  0.71048, 0.513519, 0.303986, 0.387631,
  0.0149841, 0.0914001, 0.364441, 0.134229,
  0.147308, 0.165894, 0.988495, 0.388044,
  0.445679, 0.11908, 0.00466919, 0.0897714,
  0.00891113, 0.377869, 0.531647, 0.258585,
  0.571167, 0.601746, 0.607147, 0.589803,
  0.166229, 0.663025, 0.450775, 0.357412,
  0.352112, 0.0570374, 0.607666, 0.270228,
  0.783295, 0.802582, 0.519867, 0.583348,
  0.301941, 0.875946, 0.726654, 0.562174,
  0.955872, 0.92569, 0.539337, 0.633631,
  0.142334, 0.462067, 0.235321, 0.228419,
  0.862213, 0.209595, 0.779633, 0.498077,
  0.843628, 0.996765, 0.999664, 0.930197,
  0.611481, 0.92426, 0.266205, 0.334666,
  0.297272, 0.840118, 0.0237427, 0.168081),
 nrow=20,
 ncol=4,byrow=TRUE);

indexlow=c(0.1,0.1,0.2);
indexhigh=c(0.9,0.9,0.5);

fm.fitting2additive(d, options=3, indexlow, indexhigh, option1=0, orness=c(0.1,0.7))
```

fm.fittingKinteractive

Fuzzy Measure Fitting function

Description

Estimate values of the k-interactive fuzzy measures from empirical data.

Usage

```
fm.fittingKinteractive(data, env=NULL, kadd="NA", K="NA")
```

Arguments

data	Empirical data set in pairs (x_1,y_1),(x_2,y_2),...,(x_d,y_d) where x_i in [0,1]^n is a vector containing utility values of n input criteria x_i1,x_i2,...,x_in, y_i in [0,1] is a single aggregated value given by decision makers. The data is stored as a matrix of M by n+1 elements, where M is the number of data instances, and n is the number of input criteria, the column n + 1 stores the observed aggregated value y.
env	Environment variable obtained from fm.Init(n).
kadd	Value of k-interactivity, which is used for reducing the complexity of fuzzy measures. kadd is defined as an optional argument, its default value is kadd = 2.
K	Value of FM value for sets of cardinality kadd+1, its default value is K = 0.5.

Value

output	The output is an array of size 2^n containing estimated standard fuzzy measure in binary ordering.
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
d <- matrix( c( 0.00125122, 0.563568, 0.193298, 0.164338,
 0.808716, 0.584991, 0.479858, 0.544309,
 0.350281, 0.895935, 0.822815, 0.625868,
 0.746582, 0.174103, 0.858917, 0.480347,
 0.71048, 0.513519, 0.303986, 0.387631,
 0.0149841, 0.0914001, 0.364441, 0.134229,
 0.147308, 0.165894, 0.988495, 0.388044,
 0.445679, 0.11908, 0.00466919, 0.0897714,
 0.00891113, 0.377869, 0.531647, 0.258585,
 0.571167, 0.601746, 0.607147, 0.589803,
 0.166229, 0.663025, 0.450775, 0.357412,
 0.352112, 0.0570374, 0.607666, 0.270228,
 0.783295, 0.802582, 0.519867, 0.583348,
 0.301941, 0.875946, 0.726654, 0.562174,
 0.955872, 0.92569, 0.539337, 0.633631,
 0.142334, 0.462067, 0.235321, 0.228419,
 0.862213, 0.209595, 0.779633, 0.498077,
 0.843628, 0.996765, 0.999664, 0.930197,
 0.611481, 0.92426, 0.266205, 0.334666,
 0.297272, 0.840118, 0.0237427, 0.168081),
 nrow=20,
 ncol=4,byrow=TRUE);
```

```
fm.fittingKinteractive(d,env,2,0.8)
```

fm.fittingKinteractiveAuto

Fuzzy Measure Fitting function of the k-interactive

Description

Estimate values of the k-interactive fuzzy measures from empirical data.

Usage

```
fm.fittingKinteractiveAuto(data, env=NULL, kadd="NA")
```

Arguments

data	Empirical data set in pairs (x_1,y_1),(x_2,y_2),...,(x_d,y_d) where x_i in [0,1]^n is a vector containing utility values of n input criteria x_i1,x_i2,...,x_in, y_i in [0,1] is a single aggregated value given by decision makers. The data is stored as a matrix of M by n+1 elements, where M is the number of data instances, and n is the number of input criteria, the column n + 1 stores the observed aggregated value y.
env	Environment variable obtained from fm.Init(n).
kadd	Value of k-interactivity, which is used for reducing the complexity of fuzzy measures. kadd is defined as an optional argument, its default value is kadd = 2. The constant K the value of FM value for sets of cardinality kadd+1 is computed from data.

Value

output	The output is an array of size 2^n containing estimated standard fuzzy measure in binary ordering.
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
d <- matrix( c( 0.00125122, 0.563568, 0.193298, 0.164338,
               0.808716, 0.584991, 0.479858, 0.544309,
               0.350281, 0.895935, 0.822815, 0.625868,
               0.746582, 0.174103, 0.858917, 0.480347,
               0.71048, 0.513519, 0.303986, 0.387631,
               0.0149841, 0.0914001, 0.364441, 0.134229,
               0.147308, 0.165894, 0.988495, 0.388044,
```

```

0.445679, 0.11908, 0.00466919, 0.0897714,
0.00891113, 0.377869, 0.531647, 0.258585,
0.571167, 0.601746, 0.607147, 0.589803,
0.166229, 0.663025, 0.450775, 0.357412,
0.352112, 0.0570374, 0.607666, 0.270228,
0.783295, 0.802582, 0.519867, 0.583348,
0.301941, 0.875946, 0.726654, 0.562174,
0.955872, 0.92569, 0.539337, 0.633631,
0.142334, 0.462067, 0.235321, 0.228419,
0.862213, 0.209595, 0.779633, 0.498077,
0.843628, 0.996765, 0.999664, 0.930197,
0.611481, 0.92426, 0.266205, 0.334666,
0.297272, 0.840118, 0.0237427, 0.168081),
nrow=20,
ncol=4,byrow=TRUE);
fm.fittingKinteractiveAuto(d,env,2)

```

fm.fittingKinteractiveMarginal

Fuzzy Measure Fitting function of the k-interactive using marginal representation

Description

Estimate values of the k-interactive fuzzy measures from empirical data using marginal representation.

Usage

```
fm.fittingKinteractiveMarginal(data, env=NULL, kadd="NA", K="NA", submod ="NA")
```

Arguments

data	Empirical data set in pairs (x_1,y_1),(x_2,y_2),...,(x_d,y_d) where x_i in [0,1]^n is a vector containing utility values of n input criteria x_i1,x_i2,...,x_in, y_i in [0,1] is a single aggregated value given by decision makers. The data is stored as a matrix of M by n+1 elements, where M is the number of data instances, and n is the number of input criteria, the column n + 1 stores the observed aggregated value y.
env	Environment variable obtained from fm.Init(n).
kadd	Value of k-interactive, which is used for reducing the complexity of fuzzy measures. kadd is defined as an optional argument, its default value is kadd = 2.
K	The constant K, the value of FM value for sets of cardinality kadd+1 is computed from data, default 0.5.
submod	-1 indicates supermodular FM is needed, +1 indicates submodular, 0 otherwise. Should be consistent with K and n, see manual

Value

output The output is an array of size 2^n containing estimated standard fuzzy measure in binary ordering.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
d <- matrix( c( 0.00125122, 0.563568, 0.193298, 0.164338,
  0.808716, 0.584991, 0.479858, 0.544309,
  0.350281, 0.895935, 0.822815, 0.625868,
  0.746582, 0.174103, 0.858917, 0.480347,
  0.71048, 0.513519, 0.303986, 0.387631,
  0.0149841, 0.0914001, 0.364441, 0.134229,
  0.147308, 0.165894, 0.988495, 0.388044,
  0.445679, 0.11908, 0.00466919, 0.0897714,
  0.00891113, 0.377869, 0.531647, 0.258585,
  0.571167, 0.601746, 0.607147, 0.589803,
  0.166229, 0.663025, 0.450775, 0.357412,
  0.352112, 0.0570374, 0.607666, 0.270228,
  0.783295, 0.802582, 0.519867, 0.583348,
  0.301941, 0.875946, 0.726654, 0.562174,
  0.955872, 0.92569, 0.539337, 0.633631,
  0.142334, 0.462067, 0.235321, 0.228419,
  0.862213, 0.209595, 0.779633, 0.498077,
  0.843628, 0.996765, 0.999664, 0.930197,
  0.611481, 0.92426, 0.266205, 0.334666,
  0.297272, 0.840118, 0.0237427, 0.168081),
 nrow=20,
 ncol=4,byrow=TRUE);
fm.fittingKinteractiveMarginal(d,env,2,0.6, 0)
```

fm.fittingKinteractiveMarginalMC

Fuzzy Measure Fitting function of the k-interactive using marginal representation and maximal chains method

Description

Estimate values of the k-interactive fuzzy measures from empirical data using marginal representation and maximal chains method.

Usage

```
fm.fittingKinteractiveMarginalMC(data, env=NULL, kadd="NA", K="NA", submod ="NA")
```

Arguments

data	Empirical data set in pairs (x_1,y_1),(x_2,y_2),...,(x_d,y_d) where x_i in [0,1]^n is a vector containing utility values of n input criteria x_i1,x_i2,...,x_in, y_i in [0,1] is a single aggregated value given by decision makers. The data is stored as a matrix of M by n+1 elements, where M is the number of data instances, and n is the number of input criteria, the column n + 1 stores the observed aggregated value y.
env	Environment variable obtained from fm.Init(n).
kadd	Value of k-interactivity, which is used for reducing the complexity of fuzzy measures. kadd is defined as an optional argument, its default value is kadd = 2.
K	The constant K the value of FM value for sets of cardinality kadd+1 is computed from data, default 0.5.
submod	-1 indicates supermodular FM is needed, +1 indicates submodular, 0 otherwise. Should be consistent with K and n, see manual

Value

output	The output is an array of size 2^n containing estimated standard fuzzy measure in binary ordering.
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
d <-  matrix( c( 0.00125122, 0.563568, 0.193298, 0.164338,
 0.808716, 0.584991, 0.479858, 0.544309,
 0.350281, 0.895935, 0.822815, 0.625868,
 0.746582, 0.174103, 0.858917, 0.480347,
 0.71048, 0.513519, 0.303986, 0.387631,
 0.0149841, 0.0914001, 0.364441, 0.134229,
 0.147308, 0.165894, 0.988495, 0.388044,
 0.445679, 0.11908, 0.00466919, 0.0897714,
 0.00891113, 0.377869, 0.531647, 0.258585,
 0.571167, 0.601746, 0.607147, 0.589803,
 0.166229, 0.663025, 0.450775, 0.357412,
 0.352112, 0.0570374, 0.607666, 0.270228,
 0.783295, 0.802582, 0.519867, 0.583348,
 0.301941, 0.875946, 0.726654, 0.562174,
 0.955872, 0.92569, 0.539337, 0.633631,
 0.142334, 0.462067, 0.235321, 0.228419,
 0.862213, 0.209595, 0.779633, 0.498077,
 0.843628, 0.996765, 0.999664, 0.930197,
 0.611481, 0.92426, 0.266205, 0.334666,
 0.297272, 0.840118, 0.0237427, 0.168081),
 nrow=20,
 ncol=4,byrow=TRUE);
```

```
fm.fittingKinteractiveMarginalMC(d,env,2,0.6,0)
```

fm.fittingKinteractiveMC

Fuzzy Measure Fitting function of the k-interactive using maximal chains method

Description

Estimate values of the k-interactive fuzzy measures from empirical data using maximal chains method.

Usage

```
fm.fittingKinteractiveMC(data, env=NULL, kadd="NA", K="NA")
```

Arguments

data	Empirical data set in pairs (x_1,y_1),(x_2,y_2),...,(x_d,y_d) where x_i in [0,1]^n is a vector containing utility values of n input criteria x_i1,x(i2,...,x_in), y_i in [0,1] is a single aggregated value given by decision makers. The data is stored as a matrix of M by n+1 elements, where M is the number of data instances, and n is the number of input criteria, the column n + 1 stores the observed aggregated value y.
env	Environment variable obtained from fm.Init(n).
kadd	Value of k-interactivity, which is used for reducing the complexity of fuzzy measures. kadd is defined as an optional argument, its default value is kadd = 2.
K	The constant K the value of FM value for sets of cardinality kadd+1 is computed from data, default 0.5.

Value

output	The output is an array of size 2^n containing estimated standard fuzzy measure in binary ordering.
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
d <- matrix( c( 0.00125122, 0.563568, 0.193298, 0.164338,
               0.808716, 0.584991, 0.479858, 0.544309,
               0.350281, 0.895935, 0.822815, 0.625868,
               0.746582, 0.174103, 0.858917, 0.480347,
               0.71048, 0.513519, 0.303986, 0.387631,
```

```

0.0149841, 0.0914001, 0.364441, 0.134229,
0.147308, 0.165894, 0.988495, 0.388044,
0.445679, 0.11908, 0.00466919, 0.0897714,
0.00891113, 0.377869, 0.531647, 0.258585,
0.571167, 0.601746, 0.607147, 0.589803,
0.166229, 0.663025, 0.450775, 0.357412,
0.352112, 0.0570374, 0.607666, 0.270228,
0.783295, 0.802582, 0.519867, 0.583348,
0.301941, 0.875946, 0.726654, 0.562174,
0.955872, 0.92569, 0.539337, 0.633631,
0.142334, 0.462067, 0.235321, 0.228419,
0.862213, 0.209595, 0.779633, 0.498077,
0.843628, 0.996765, 0.999664, 0.930197,
0.611481, 0.92426, 0.266205, 0.334666,
0.297272, 0.840118, 0.0237427, 0.168081),
nrow=20,
ncol=4,byrow=TRUE);
fm.fittingKinteractiveMC(d,env,2,0.6)

```

fm.fittingKmaxitive *Fuzzy Measure Fitting function of the k-maxitive*

Description

Estimate values of the k-maxitive fuzzy measures from empirical data.

Usage

```
fm.fittingKmaxitive(data, env=NULL, kadd="NA")
```

Arguments

data	Empirical data set in pairs (x_1,y_1),(x_2,y_2),...,(x_d,y_d) where x_i in [0,1]^n is a vector containing utility values of n input criteria x_i1,x_i2,...,x_in, y_i in [0,1] is a single aggregated value given by decision makers. The data is stored as a matrix of M by n+1 elements, where M is the number of data instances, and n is the number of input criteria, the column n + 1 stores the observed aggregated value y.
env	Environment variable obtained from fm.Init(n).
kadd	Value of k-maxitivity, which is used for reducing the complexity of fuzzy measures. kadd is defined as an optional argument, its default value is kadd = n.

Value

output	The output is an array of size 2^n containing estimated standard fuzzy measure in binary ordering.
---------------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
d <- matrix( c( 0.00125122, 0.563568, 0.193298, 0.164338,
               0.808716, 0.584991, 0.479858, 0.544309,
               0.350281, 0.895935, 0.822815, 0.625868,
               0.746582, 0.174103, 0.858917, 0.480347,
               0.71048, 0.513519, 0.303986, 0.387631,
               0.0149841, 0.0914001, 0.364441, 0.134229,
               0.147308, 0.165894, 0.988495, 0.388044,
               0.445679, 0.11908, 0.00466919, 0.0897714,
               0.00891113, 0.377869, 0.531647, 0.258585,
               0.571167, 0.601746, 0.607147, 0.589803,
               0.166229, 0.663025, 0.450775, 0.357412,
               0.352112, 0.0570374, 0.607666, 0.270228,
               0.783295, 0.802582, 0.519867, 0.583348,
               0.301941, 0.875946, 0.726654, 0.562174,
               0.955872, 0.92569, 0.539337, 0.633631,
               0.142334, 0.462067, 0.235321, 0.228419,
               0.862213, 0.209595, 0.779633, 0.498077,
               0.843628, 0.996765, 0.999664, 0.930197,
               0.611481, 0.92426, 0.266205, 0.334666,
               0.297272, 0.840118, 0.0237427, 0.168081),
               nrow=20,
               ncol=4,byrow=TRUE);
fm.fittingKmaxitive(d,env,2)
```

fm.fittingKtolerant *Fuzzy Measure Fitting function of the k-tolerant*

Description

Estimate values of the k-tolerant fuzzy measures from empirical data.

Usage

```
fm.fittingKtolerant(data, env=NULL, kadd="NA")
```

Arguments

data	Empirical data set in pairs (x_1,y_1),(x_2,y_2),...,(x_d,y_d) where x_i in [0,1]^n is a vector containing utility values of n input criteria x_i1,x_i2,...,x_in, y_i in [0,1] is a single aggregated value given by decision makers. The data is stored as a matrix of M by n+1 elements, where M is the number of data instances, and n is the number of input criteria, the column n + 1 stores the observed aggregated value y.
-------------	--

env	Environment variable obtained from fm.Init(n).
kadd	Value of k-tolerance, which is used for reducing the complexity of fuzzy measures. kadd is defined as an optional argument, its default value is kadd = n.

Value

output	The output is an array of size 2^n containing estimated standard fuzzy measure in binary ordering.
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
d <- matrix( c( 0.00125122, 0.563568, 0.193298, 0.164338,
 0.808716, 0.584991, 0.479858, 0.544309,
 0.350281, 0.895935, 0.822815, 0.625868,
 0.746582, 0.174103, 0.858917, 0.480347,
 0.71048, 0.513519, 0.303986, 0.387631,
 0.0149841, 0.0914001, 0.364441, 0.134229,
 0.147308, 0.165894, 0.988495, 0.388044,
 0.445679, 0.11908, 0.00466919, 0.0897714,
 0.00891113, 0.377869, 0.531647, 0.258585,
 0.571167, 0.601746, 0.607147, 0.589803,
 0.166229, 0.663025, 0.450775, 0.357412,
 0.352112, 0.0570374, 0.607666, 0.270228,
 0.783295, 0.802582, 0.519867, 0.583348,
 0.301941, 0.875946, 0.726654, 0.562174,
 0.955872, 0.92569, 0.539337, 0.633631,
 0.142334, 0.462067, 0.235321, 0.228419,
 0.862213, 0.209595, 0.779633, 0.498077,
 0.843628, 0.996765, 0.999664, 0.930197,
 0.611481, 0.92426, 0.266205, 0.334666,
 0.297272, 0.840118, 0.0237427, 0.168081),
 nrow=20,
 ncol=4,byrow=TRUE);
fm.fittingKtolerant(d,env,2)
```

Description

Estimate values of the Mobius fuzzy measures from empirical data.

Usage

```
fm.fittingMob(data, env=NULL ,kadd="NA")
```

Arguments

data	Empirical data set in pairs (x_1,y_1),(x_2,y_2),...,(x_d,y_d) where x_i in [0,1]^n is a vector containing utility values of n input criteria x_i1,x_i2,...,x_in, y_i in [0,1] is a single aggregated value given by decision makers. The data is stored as a matrix of M by n+1 elements, where M is the number of data instances, and n is the number of input criteria, the column n + 1 store the observed aggregating value y.
env	Environment variable obtained from fm.Init(n).
kadd	value of k-additivity, which is used for reducing the complexity of fuzzy measures. kadd is defined as an optional argument, its defaultvalue is kadd = n.

Value

output	The output is an array of size 2^n containing estimated Mobius fuzzy measure in binary ordering.
---------------	--

Note

The fit might not be perfect, and not all the constraints can be fully met.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
d <- matrix( c( 0.00125122, 0.563568, 0.193298, 0.164338,
 0.808716, 0.584991, 0.479858, 0.544309,
 0.350281, 0.895935, 0.822815, 0.625868,
 0.746582, 0.174103, 0.858917, 0.480347,
 0.71048, 0.513519, 0.303986, 0.387631,
 0.0149841, 0.0914001, 0.364441, 0.134229,
 0.147308, 0.165894, 0.988495, 0.388044,
 0.445679, 0.11908, 0.00466919, 0.0897714,
 0.00891113, 0.377869, 0.531647, 0.258585,
 0.571167, 0.601746, 0.607147, 0.589803,
 0.166229, 0.663025, 0.450775, 0.357412,
 0.352112, 0.0570374, 0.607666, 0.270228,
 0.783295, 0.802582, 0.519867, 0.583348,
 0.301941, 0.875946, 0.726654, 0.562174,
 0.955872, 0.92569, 0.539337, 0.633631,
 0.142334, 0.462067, 0.235321, 0.228419,
 0.862213, 0.209595, 0.779633, 0.498077,
 0.843628, 0.996765, 0.999664, 0.930197,
 0.611481, 0.92426, 0.266205, 0.334666,
 0.297272, 0.840118, 0.0237427, 0.168081),
```

```

nrow=20,
ncol=4,byrow=TRUE);
env<-fm.Init(3)
fm.fittingMob(d,env)

```

fm.fittingOWA*Symmetric Fuzzy Measure Fitting function***Description**

Estimate values of the symmetric fuzzy measures from empirical data. The resulting Choquet integral is the OWA function.

Usage

```
fm.fittingOWA(data, env=NULL)
```

Arguments

data	Empirical data set in pairs $(x_1, y_1), (x_2, y_2), \dots, (x_d, y_d)$ where $x_i \in [0,1]^n$ is a vector containing utility values of n input criteria $x_{i1}, x_{i2}, \dots, x_{in}$, $y_i \in [0,1]$ is a single aggregated value given by decision makers. The data is stored as a matrix of M by $n+1$ elements, where M is the number of data instances, and n is the number of input criteria, the column $n + 1$ stores the observed aggregated value y .
env	Environment variable obtained from fm.Init(n).

Value

output The output is an array of size n containing estimated OWA coefficients.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```

env<-fm.Init(3)
d <- matrix( c( 0.00125122, 0.563568, 0.193298, 0.164338,
 0.808716, 0.584991, 0.479858, 0.544309,
 0.350281, 0.895935, 0.822815, 0.625868,
 0.746582, 0.174103, 0.858917, 0.480347,
 0.71048, 0.513519, 0.303986, 0.387631,
 0.0149841, 0.0914001, 0.364441, 0.134229,
 0.147308, 0.165894, 0.988495, 0.388044,
 0.445679, 0.11908, 0.00466919, 0.0897714,
 0.00891113, 0.377869, 0.531647, 0.258585,
 0.571167, 0.601746, 0.607147, 0.589803,

```

```

0.166229, 0.663025, 0.450775, 0.357412,
0.352112, 0.0570374, 0.607666, 0.270228,
0.783295, 0.802582, 0.519867, 0.583348,
0.301941, 0.875946, 0.726654, 0.562174,
0.955872, 0.92569, 0.539337, 0.633631,
0.142334, 0.462067, 0.235321, 0.228419,
0.862213, 0.209595, 0.779633, 0.498077,
0.843628, 0.996765, 0.999664, 0.930197,
0.611481, 0.92426, 0.266205, 0.334666,
0.297272, 0.840118, 0.0237427, 0.168081),
nrow=20,
ncol=4,byrow=TRUE);
fm.fittingOWA(d,env)

```

fm.fittingWAM*Additive Fuzzy Measure Fitting function***Description**

Estimate values of an additive fuzzy measure from empirical data. In this case the Choquet integral is the weighted arithmetic mean WAM.

Usage

```
fm.fittingWAM(data, env=NULL)
```

Arguments

data	Empirical data set in pairs $(x_1, y_1), (x_2, y_2), \dots, (x_d, y_d)$ where x_i in $[0,1]^n$ is a vector containing utility values of n input criteria $x_{i1}, x_{i2}, \dots, x_{in}$, y_i in $[0,1]$ is a single aggregated value given by decision makers. The data is stored as a matrix of M by $n+1$ elements, where M is the number of data instances, and n is the number of input criteria, the column $n + 1$ stores the observed aggregated value y .
env	Environment variable obtained from fm.Init(n).

Value

output	The output is an array of size n containing estimated weighting vector of WAM.
---------------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```

env<-fm.Init(3)
d <- matrix( c( 0.00125122, 0.563568, 0.193298, 0.164338,
               0.808716, 0.584991, 0.479858, 0.544309,
               0.350281, 0.895935, 0.822815, 0.625868,
               0.746582, 0.174103, 0.858917, 0.480347,
               0.71048, 0.513519, 0.303986, 0.387631,
               0.0149841, 0.0914001, 0.364441, 0.134229,
               0.147308, 0.165894, 0.988495, 0.388044,
               0.445679, 0.11908, 0.00466919, 0.0897714,
               0.00891113, 0.377869, 0.531647, 0.258585,
               0.571167, 0.601746, 0.607147, 0.589803,
               0.166229, 0.663025, 0.450775, 0.357412,
               0.352112, 0.0570374, 0.607666, 0.270228,
               0.783295, 0.802582, 0.519867, 0.583348,
               0.301941, 0.875946, 0.726654, 0.562174,
               0.955872, 0.92569, 0.539337, 0.633631,
               0.142334, 0.462067, 0.235321, 0.228419,
               0.862213, 0.209595, 0.779633, 0.498077,
               0.843628, 0.996765, 0.999664, 0.930197,
               0.611481, 0.92426, 0.266205, 0.334666,
               0.297272, 0.840118, 0.0237427, 0.168081),
               nrow=20,
               ncol=4,byrow=TRUE);
fm.fittingWAM(d,env)

```

fm.fm_arraysize*Function for returning the length of the array*

Description

Returns the length of the array of values of k-interactive fuzzy measures. Useful for reserving memory.

Usage

```
fm.fm_arraysize(env = NULL, kint = "NA")
```

Arguments

env	Environment variable obtained from fm.Init(n).
kint	Interactive fuzzy measure. $0 < \text{kint} \leq n$

Value

output	The outputs is the length of the array of values of k-interactive fuzzy measures
---------------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
arraylength <- fm.fm_arraysize(env,1)
```

fm.Free

FreeSparseFM function

Description

Frees the memory previously allocated in env.

Usage

```
fm.Free(env)
```

Arguments

env Structure required for auxiliary data. It is obtained from fm.Init(n).

Value

output Frees the memory previously allocated in envsp.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n<-3
env <- fm.Init(n)
env<-fm.Free(env)
env
```

fm.FreeSparseFM *FreeSparseFM function*

Description

Frees the memory previously allocated in envsp.

Usage

```
fm.FreeSparseFM(envsp)
```

Arguments

envsp	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).
-------	--

Value

output	Frees the memory previously allocated in envsp.
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n<-3
envsp <- fm.PrepareSparseFM(n, vector(), vector())
envsp <- fm.FreeSparseFM(envsp)
envsp <- fm.PrepareSparseFM(n, c(0.2,0.4,0.1), c(2,1,2,2,1,3,3,1,2,3))
envsp <- fm.FreeSparseFM(envsp)
envsp
```

fm.FuzzyMeasureFitLP *Fuzzy Measure Fitting function.*

Description

Estimate values of the fuzzy measures from empirical data. The result is an array containing the values of a standard fuzzy measure in binary ordering. kadd defines the complexity of fuzzy measure. If kadd is not provided, its default value is equal to the number of inputs.

Usage

```
fm.FuzzyMeasureFitLP(data, env=NULL, kadd="NA",
                      options=0, indexlow=(NULL), indexhigh=(NULL) , option1=0, orness=(NULL))
```

Arguments

data	Empirical data set in pairs $(x_1, y_1), (x_2, y_2), \dots, (x_d, y_d)$ where $x_i \in [0, 1]^n$ is a vector contains utility values of n input criteria x_i1, x_i2, \dots, x_in , $y_i \in [0, 1]$ is a single aggregated value given by decision makers. The data is stored as a matrix of M by $n+1$ elements, where M is the number of data instances, and n is the number of input criteria, the column $n + 1$ store the observed aggregating value y .
env	Environment variable obtained from <i>fm.Init(n)</i> .
kadd	Value of k-additivity, which is used for reducing the complexity of fuzzy measures. <i>kadd</i> is defined as an optional argument, its default value is <i>kadd</i> = n . <i>kadd</i> is k in k -additive f-measure, $1 < kadd < n+1$; if <i>kdd=n</i> - f.m. is unrestricted
options	Options default value is 0. 1 - lower bounds on Shapley values supplied in <i>indexlow</i> , 2 - upper bounds on Shapley values supplied in <i>indexhigh</i> , 3 - lower and upper bounds on Shapley values supplied in <i>indexlow</i> and <i>indexhigh</i> , 4 - lower bounds on all interaction indices supplied in <i>indexlow</i> , 5 - upper bounds on all interaction indices supplied in <i>indexhigh</i> , 6 - lower and upper bounds on all interaction indices supplied in <i>indexlow</i> and <i>indexhigh</i> . All these value will be treated as additional constraints in the LP.
indexlow	Array of size n (<i>options</i> =1,2,3) or m (<i>options</i> =4,5,6) containing the lower bounds on the Shapley values or interaction indices
indexhigh	Array of size n (<i>options</i> =1,2,3) or m (<i>options</i> =4,5,6) containing the upper bounds on the Shapley values or interaction indices
option1	If the value is 1, the interval of orness values will be fitted (and the desired low and high orness values should be provided). If 0, no additional orness constraints.
orness	Array of size 2, for example <i>c(0.1,1)</i>

Value

output	The output is an array of size 2^n containing estimated standard fuzzy measure in binary ordering.
---------------	--

Note

The fit might not be perfect, and not all the constraints can be fully met.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
d <- matrix( c( 0.00125122, 0.563568, 0.193298, 0.164338,
 0.808716, 0.584991, 0.479858, 0.544309,
 0.350281, 0.895935, 0.822815, 0.625868,
 0.746582, 0.174103, 0.858917, 0.480347,
 0.71048, 0.513519, 0.303986, 0.387631,
```

```

0.0149841, 0.0914001, 0.364441, 0.134229,
0.147308, 0.165894, 0.988495, 0.388044,
0.445679, 0.11908, 0.00466919, 0.0897714,
0.00891113, 0.377869, 0.531647, 0.258585,
0.571167, 0.601746, 0.607147, 0.589803,
0.166229, 0.663025, 0.450775, 0.357412,
0.352112, 0.0570374, 0.607666, 0.270228,
0.783295, 0.802582, 0.519867, 0.583348,
0.301941, 0.875946, 0.726654, 0.562174,
0.955872, 0.92569, 0.539337, 0.633631,
0.142334, 0.462067, 0.235321, 0.228419,
0.862213, 0.209595, 0.779633, 0.498077,
0.843628, 0.996765, 0.999664, 0.930197,
0.611481, 0.92426, 0.266205, 0.334666,
0.297272, 0.840118, 0.0237427, 0.168081),
nrow=20,
ncol=4,byrow=TRUE);
env<-fm.Init(3)
fm.FuzzyMeasureFitLP(d,env)
indexlow=c(0.1,0.1,0.2);
indexhigh=c(0.9,0.9,0.5);
fm.FuzzyMeasureFitLP(d,env, kadd=2, indexlow, indexhigh,
options=3, option1=1, orness=c(0.1,0.7))

```

fm.FuzzyMeasureFitLPMob

Mobius Fuzzy Measure Fitting function, R wrapper for FuzzyMeasureFitLP() in fuzzymeasurefit.cpp

Description

Estimate values of the Mobius fuzzy measures from empirical data. The result is an array containing the values of the fuzzy measure in Mobius, ordered according to set cardinalities. kadd defines the complexity of fuzzy measure. if kadd is not provided, its default value is equal to the number of inputs.

Usage

```
fm.FuzzyMeasureFitLPMob(data, env=NULL, kadd="NA",
options=0, indexlow=(NULL), indexhigh=(NULL) , option1=0, orness=(NULL))
```

Arguments

data	Empirical data set in pairs (x_1,y_1),(x_2,y_2),...,(x_d,y_d) where x_i in [0,1]^n is a vector contains utility values of n input criteria x_i1,x_i2,...,x_in, y_i in [0,1] is a single aggregated value given by decision makers. The data is stored as a matrix of M by n+1 elements, where M is the number of data instances, and n is the number of input criteria, the column n + 1 store the observed aggregating value y.
------	--

env	Environment variable obtained from fm.Init(n).
kadd	Value of k-additivity, which is used for reducing the complexity of fuzzy measures. kadd is defined as an optional argument, its default value is kadd = n. kadd is k in k-additive f-measure, $1 < \text{kadd} < n+1$; if kadd=n - f.m. is unrestricted
options	Options default value is 0. 1 - lower bounds on Shapley values supplied in indexlow, 2 - upper bounds on Shapley values supplied in indexhigh, 3 - lower and upper bounds on Shapley values supplied in indexlow and indexhigh, 4 - lower bounds on all interaction indices supplied in indexlow, 5 - upper bounds on all interaction indices supplied in indexhigh, 6 - lower and upper bounds on all interaction indices supplied in indexlow and indexhigh. All these value will be treated as additional constraints in the LP.
indexlow	Array of size n (options =1,2,3) or m (options=4,5,6) containing the lower bounds on the Shapley values or interaction indices
indexhigh	Array of size n (options =1,2,3) or m (options=4,5,6) containing the upper bounds on the Shapley values or interaction indices
option1	If the value is 1, the interval of orness values will be fitted (and the desired low and high orness values should be provided). If 0, no additional orness constraints.
orness	Array of size 2, for example c(0.1,1)

Value

output	The output is an array of size 2^n containing estimated Mobius fuzzy measure in binary ordering.
--------	--

Note

The fit might not be perfect, and not all the constraints can be fully met.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
d <- matrix( c( 0.00125122, 0.563568, 0.193298, 0.164338,
 0.808716, 0.584991, 0.479858, 0.544309,
 0.350281, 0.895935, 0.822815, 0.625868,
 0.746582, 0.174103, 0.858917, 0.480347,
 0.71048, 0.513519, 0.303986, 0.387631,
 0.0149841, 0.0914001, 0.364441, 0.134229,
 0.147308, 0.165894, 0.988495, 0.388044,
 0.445679, 0.11908, 0.00466919, 0.0897714,
 0.00891113, 0.377869, 0.531647, 0.258585,
 0.571167, 0.601746, 0.607147, 0.589803,
 0.166229, 0.663025, 0.450775, 0.357412,
 0.352112, 0.0570374, 0.607666, 0.270228,
 0.783295, 0.802582, 0.519867, 0.583348,
 0.301941, 0.875946, 0.726654, 0.562174,
```

```
0.955872, 0.92569, 0.539337, 0.633631,
0.142334, 0.462067, 0.235321, 0.228419,
0.862213, 0.209595, 0.779633, 0.498077,
0.843628, 0.996765, 0.999664, 0.930197,
0.611481, 0.92426, 0.266205, 0.334666,
0.297272, 0.840118, 0.0237427, 0.168081),
nrow=20,
ncol=4,byrow=TRUE);
env<-fm.Init(3)
fm.FuzzyMeasureFitLPMob(d,env)
indexlow=c(0.1,0.1,0.2);
indexhigh=c(0.9,0.9,0.5);
fm.FuzzyMeasureFitLPMob(d,env, kadd=2, indexlow, indexhigh,
options=3, optionl=1, orness=c(0.1,0.7))
```

fm.generate_antibuoant

Function for generating one antibuoant random fuzzy measure

Description

Generates one antibuoant random fuzzy measure in standard representation.

Usage

```
fm.generate_antibuoant(env = NULL)
```

Arguments

env Environment variable obtained from fm.Init(n).

Value

output The output is one antibuoant random fuzzy measure.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fuzzymeasures <- fm.generate_antibuoant(env)
fuzzymeasures
```

fm.generate_balanced *Function for random generation of balanced fuzzy measures in standard representation*

Description

Generate several balanced random fuzzy measures in standard representation.

Usage

```
fm.generate_balanced(num, env=NULL)
```

Arguments

num	Generates num random fuzzy measures stored in an array v of length num * 2n.
env	Environment variable obtained from fm.Init(n).

Value

output	The output are several random fuzzy measures containing in an array v of length num * 2n
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fuzzymeasures <- fm.generate_balanced(2, env)
fuzzymeasures
```

fm.generate_belief *Function for random generation of belief fuzzy measures in standard representation*

Description

Generate several random k-additive belief measures in Mobius representation.

Usage

```
fm.generate_belief(num, kadd, env=NULL)
```

Arguments

num	Generates num random belief measures stored in an array Mob of length num * fm_arraysize_kadd(n, kadd).
kadd	k-additivity
env	Environment variable obtained from fm.Init(n).

Value

output	The output are several random belief measures containing in an array v of length num * fm_a
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(5)
belief <- fm.generate_belief(2, 3, env)
# 2 3-additive measures with n=5
belief
```

fm.generate_fmconvex_tsort

Function for generating convex fuzzy measures

Description

Generates num convex random fuzzy measures stored consecutively in cardinality ordering in the output array.

Usage

```
fm.generate_fmconvex_tsort(num, kint, markov, option, K, env = NULL)
```

Arguments

num	Several random fuzzy measures stored in cardinality ordering in the array v (num is their number)
kint	Interactive fuzzy measure. 0 < kint <= n
markov	Number of Markov steps to take, the randomness increases with that number
option	Option = 1 employs internal rejection method to improve uniformity, but for n > 5 is not essential
K	K is the constant in k-interactive fuzzy measures
env	Environment variable obtained from fm.Init(n).

Value

- output** The output is the generation of num convex random fuzzy measures stored consecutively in cardinality ordering in the array v

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fuzzymeasures <- fm.generate_fmconvex_tsort(1,3,1000,0,1, env)
```

fm.generate_fm_2additive

Function for generating 2-additive fuzzy measures in Möbius representation

Description

Generates num random 2-additive fuzzy measures in Möbius representation.

Usage

```
fm.generate_fm_2additive(num, n)
```

Arguments

- num** Generates num random fuzzy measures stored consecutively in cardinality ordering in the array v.
- n** Number of inputs

Value

- output** The output are random fuzzy measures, it contains singletons and pairs but no emptyset

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
num <- 2
n <- 5
fuzzymeasures <- fm.generate_fm_2additive(num,n)
fuzzymeasures$V
fuzzymeasures$len
```

`fm.generate_fm_2additive_concave`

Function for generating 2additive concave fuzzy measures.

Description

Generates num 2-additive concave (supermodular) fuzzy measures for n inputs.

Usage

```
fm.generate_fm_2additive_concave(num, n)
```

Arguments

num	Generated num concave random fuzzy measures stored consecutively in cardinality ordering in the array v
n	Number of inputs

Value

output	The output is the length of the part of the array v allocated for each fuzzy measure, and the array with singletons and pairs in Mobius representation
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
fuzzymeasures <- fm.generate_fm_2additive_concave(10,20)
```

fm.generate_fm_2additive_convex

Function for generating 2-additive convex fuzzy measures

Description

Generates num 2-additive convex (supermodular) fuzzy measures for n inputs.

Usage

```
fm.generate_fm_2additive_convex(num, n)
```

Arguments

num	Generates num convex random fuzzy measures stored consecutively in cardinality ordering in the array v
n	Number of inputs

Value

output	The output is the length of the part of the array v allocated for each fuzzy measure, and the array with singletons and pairs in Mobius representation
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
addconvex <- fm.generate_fm_2additive_convex(5,20)
```

fm.generate_fm_2additive_convex_sparse

Function for generating 2additive convex numbers in sparse representation

Description

Generates a random 2-additive supermodular fuzzy measure in sparse representation.

Usage

```
fm.generate_fm_2additive_convex_sparse(n, envsp = NULL)
```

Arguments

n	Number of inputs
envsp	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

output	The output are 2-additive supermodular fuzzy measure in sparse representation
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 5
tups<-vector()
tupsidx<-vector()
envsp <- fm.PrepareSparseFM(n, tups,tupsidx)
envsp <- fm.generate_fm_2additive_convex_sparse(n, envsp)
envsp
envsp <- fm.FreeSparseFM(envsp)
```

fm.generate_fm_2additive_convex_withsomeindependent

Function for generating 2additive convex fuzzy measures with some independent inputs

Description

Generates num 2-additive convex (supermodular) fuzzy measures for n inputs. Some of the interaction indices are set to 0 (independence).

Usage

```
fm.generate_fm_2additive_convex_withsomeindependent(num, n)
```

Arguments

num	Generates num convex random fuzzy measures stored consecutively in cardinality ordering in the array
n	Number of inputs

Value

output	The output is the length of the part of the array v allocated for each fuzzy measure, and the array with singletons and pairs in Mobius representation
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
addconvex <- fm.generate_fm_2additive_convex_withsomeindependent(5, 20)
```

fm.generate_fm_2additive_randomwalk2

Function for generating random 2-additive fuzzy measures in Möbius representation by using random walk.

Description

Generate a random 2-additive fuzzy measures in Möbius representation by using random walk.

Usage

```
fm.generate_fm_2additive_randomwalk2(num, n, markov, option, step, Fn)
```

Arguments

num	Generated num random fuzzy measures stored consecutively in cardinality ordering in the array v.
n	Number of inputs.
markov	Number of Markov steps to take, the randomness increases with that number.
option	Not used, reserved for future use.
step	The maximum size of random steps (with respect to each value). The actual step is a random value up to Step.
Fn	The callback function to verify any additional conditions on generated FM. Provided by the user or NULL.

Value

output	The output are random 2-additive fuzzy measure, it contains singletons and pairs but no emptyset.
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
num <- 10
n <- 5
fuzzymeasures <- fm.generate_fm_2additive_randomwalk2(num, n, 1000, 0, 0.001, NULL)
```

fm.generate_fm_kadditive_convex_sparse
Generate kadditive convex sparse fuzzy measures

Description

Generates a random k-additive Belief fuzzy measure in sparse representation

Usage

```
fm.generate_fm_kadditive_convex_sparse(n, kadd, nonzero, envsp = NULL)
```

Arguments

n	Inputs length. (n inputs)
kadd	kadd is the value of k-additivity, which is used for reducing the complexity of fuzzy measures. default value is kadd = n. $1 < \text{kadd} < n+1$; if kdd=n - f.m. is unrestricted
nonzero	Values stored and indexed in the respective arrays which are part of the structure
envsp	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

output The output is k-additive Belief fuzzy measure in sparse representation

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 5
tups<-vector()
tupsidx<-vector()
envsp <- fm.PrepareSparseFM(n, tups,tupsidx)
envsp <- fm.generate_fm_kadditive_convex_sparse(n,4,10, envsp)
envsp
envsp <- fm.FreeSparseFM(envsp)
```

fm.generate_fm_kinteractivedualconcave

Function for generating k-interactive dual concave fuzzy measures in Mobius representation

Description

Generates num k-interactive dual concave fuzzy measures in Mobius representation using random walk of length markov of stepsize step

Usage

```
fm.generate_fm_kinteractivedualconcave(num, n, kadd, markov, step, Fn, env)
```

Arguments

num	Generated num random fuzzy measures stored consecutively in cardinality ordering in the array v.
n	Number of inputs.
kadd	kadd is the value of k-additivity, which is used for reducing the complexity of fuzzy measures. default value is kadd = n. $1 < \text{kadd} < n+1$; if kdd=n - f.m. is unrestricted.
markov	Number of Markov steps to take, the randomness increases with that number.
step	The maximum size of random steps (with respect to each value). The actual step is a random value up to Step.
Fn	The callback function to verify any additional conditions on generated FM. Provided by the user.
env	Environment variable obtained from fm.Init(n).

Value

output	The output are k-interactive dual concave fuzzy measures in Mobius representation
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(4)
step <- 0.001
Fn <- NULL

fuzzymeasures <- fm.generate_fm_kinteractivedualconcave(10, 4, 2, 1000, step, Fn, env)
fuzzymeasures
```

fm.generate_fm_kinteractivedualconvex

Function for generating several k-interactive dual convex fuzzy measures in Möbius representation

Description

Generates num k-interactive dual convex fuzzy measures in Möbius representation using random walk of length markov of stepsize step.

Usage

```
fm.generate_fm_kinteractivedualconvex(num, n, kadd, markov, step, Fn, env)
```

Arguments

num	Generated num random fuzzy measures stored consecutively in cardinality ordering in the array v.
n	Number of inputs.
kadd	kadd is the value of k-additivity, which is used for reducing the complexity of fuzzy measures. default value is kadd = n. $1 < \text{kadd} < n+1$; if kdd=n - f.m. is unrestricted.
markov	Number of Markov steps to take, the randomness increases with that number.
step	The maximum size of random steps (with respect to each value). The actual step is a random value up to Step.
Fn	The callback function to verify any additional conditions on generated FM. Provided by the user.
env	Environment variable obtained from fm.Init(n).

Value

output	The output are several k-interactive dual convex fuzzy measures in Möbius representation
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(4)
step <- 0.0001
Fn <- NULL

fuzzymeasures <- fm.generate_fm_kinteractivedualconvex(10, 4, 2, 1000, step, Fn, env)
fuzzymeasures

env<-fm.Free(env)
```

fm.generate_fm_minplus

Generate randomly fuzzy measures

Description

Generate several random fuzzy measures (num is their number) stored in cardinality ordering in the array v using minimals_plus method.

Usage

```
fm.generate_fm_minplus(num, kint, markov, option, K, env = NULL)
```

Arguments

num	Generated num random fuzzy measures stored consecutively in cardinality ordering in the array
kint	Interactive fuzzy measure. $0 < \text{kint} \leq n$
markov	Number of Markov steps to take, the randomness increases with that number
option	Option = 1 employs internal rejection method to improve uniformity, but for $n > 5$ is not essential
K	K is the constant in k-interactive fuzzy measures
env	Environment variable obtained from fm.Init(n).

Value

output The output is generate several random fuzzy measures

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fuzzymeasures <- fm.generate_fm_minplus(10,3,1000,0,0.7, env)
fuzzymeasures
```

fm.generate_fm_randomwalk*Function for generating several k-additive fuzzy measure*

Description

Generates num k-additive fuzzy measures in the standard or Mobius representation using random walk of length markov of stepsize step.

Usage

```
fm.generate_fm_randomwalk(num, n, kadd, markov, option, step, Fn, env)
```

Arguments

num	Generated num random fuzzy measures stored consecutively in standard or cardinality ordering in the array v.
n	Number of inputs
kadd	kadd is the value of k-additivity, which is used for reducing the complexity of fuzzy measures. default value is kadd = n. $1 < \text{kadd} < n+1$; if $\text{kadd}=n$ - f.m. is unrestricted. The parameter kadd only matters for options 3 and 5
markov	Number of Markov steps to take, the randomness increases with that number.
option	Option = 0 - normal, 1 convex (supermodular), 2 antibuoyant, 3 kadditive , 4 belief measure, 5 kadditive convex. The measure generated is in standard representation fo all options except 3,5. The parameter kadd only matters for options 3 and 5. In that case the measure is in more compact Mobius representation.
step	The maximum size of random steps (with respect to each value). The actual step is a random value up to Step.
Fn	The callback function to verify any additional conditions on generated FM. Provided by the user. if not NULL, is a callback function to perform additional check at every Markov step of the current set function, i.e., any extra conditions
env	Environment variable obtained from fm.Init(n).

Value

output	The output is named list with the first element v being the fuzzy measure and the second being the length of the array containing it
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```

Fn <- function(n,v){
  out <- 0.0

  for(i in 1:n) out<- out+v[i];

  if(out>1) {
    return(0)
  } else
    return(1)
}

env<-fm.Init(3)
step <- 0.0010
Option <- 3
n <- 3

fuzzymeasures <- fm.generate_fm_randomwalk(2, 3, 2, 1000, Option, step, Fn, env)
print(fuzzymeasures)
print(fuzzymeasures$length)

```

fm.generate_fm_sorting

Function for random generation of fuzzy measures in standard representation

Description

Generate several random fuzzy measures in standard representation

Usage

```
fm.generate_fm_sorting(num, markov, option, env = NULL)
```

Arguments

num	Generates num random fuzzy measures stored in an array v of length num * 2n.
markov	Number of Markov steps to take, the randomness increases with that number.
option	Option = 1 employs internal rejection method to improve uniformity, but for n > 5 is not essential.
env	Environment variable obtained from fm.Init(n).

Value

output	The output are several random fuzzy measures containing in an array v of length num * 2n
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
markovsteps <- 100
fuzzymeasures <- fm.generate_fm_sorting(5, markovsteps, 0, env)
fuzzymeasures
```

fm.generate_fm_tsort *Function for random generation of fuzzy measures*

Description

Generate several random fuzzy measures (num is their number) stored in cardinality ordering in the array v using topological sort.

Usage

```
fm.generate_fm_tsort(num, kint, markov, option, K, env = NULL)
```

Arguments

num	Generated num random fuzzy measures stored consecutively in cardinality ordering in the array
kint	Interactive fuzzy measure. $0 < kint \leq n$
markov	Number of Markov steps to take, the randomness increases with that number
option	Option = 1 employs internal rejection method to improve uniformity, but for $n > 5$ is not essential
K	K is the constant in k-interactive fuzzy measures
env	Environment variable obtained from fm.Init(n).

Value

output The output is generate several random fuzzy measures

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fuzzymeasures <- fm.generate_fm_tsort(10,3,1000,0,0.7, env)
fuzzymeasures
```

fm.get_num_tuples *Function for exporting number of tuples*

Description

Returns the number of tuples.

Usage

```
fm.get_num_tuples(envsp=NULL)
```

Arguments

envsp	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).
-------	--

Value

output	The output is the number of tuples.
--------	-------------------------------------

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
envsp <- fm.PrepareSparseFM(n, vector(), vector())
envsp <- fm.add_singletons_sparse(c(0.2,0.1,0.2),envsp)
envsp <- fm.add_tuple_sparse(c(1,2,3),0.4,envsp);
fm.get_num_tuples(envsp)

envsp <- fm.FreeSparseFM(envsp)
```

fm.get_sizearray_tuples

Function for exporting the size of the array of tuples

Description

Returns the length of the array of tuples.

Usage

```
fm.get_sizearray_tuples(envsp=NULL)
```

Arguments

`envsp` Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

`output` The output is the length of the array of tuples.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
envsp <- fm.PrepareSparseFM(n, vector(), vector())
envsp <- fm.add_singletons_sparse(c(0.2,0.1,0.2),envsp)
envsp <- fm.add_tuple_sparse(c(1,2,3),0.4,envsp);
fm.get_sizearray_tuples(envsp)

envsp <- fm.FreeSparseFM(envsp)
```

fm.Init

Initialisation function

Description

This function initialises the internal structures which makes computations faster. The structures are saved in the output environment variable, which should be subsequently passed to other functions. Several environment variables (for different dimensions) can be initialised at the same time.

Usage

```
fm.Init(n1)
```

Arguments

`n1` The number of variables.

Value

`output` The ouput is the enviromnet variable containing the internal structures.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
```

fm.Interaction	<i>Interaction Index computation function</i>
----------------	---

Description

Calculates all the interaction indices of input criteria for standard fuzzy measure.

Usage

```
fm.Interaction(v, env)
```

Arguments

v	Fuzzy measure value in standard representation
env	Environment variable obtained from fm.Init(n).

Value

output	The output is a matrix, whose first column stores the interaction index values, and the second column stores the indices of criteria in coalitions.
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.Interaction(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1), env)
```

fm.InteractionB	<i>Banzhaf Interaction Index computation function</i>
-----------------	---

Description

Calculates all the Banzhaf Interaction indices of input criteria for a standard fuzzy measure.

Usage

```
fm.InteractionB(v, env)
```

Arguments

v	Fuzzy measure value in standard representation
env	Environment variable obtained from fm.Init(n).

Value

output	The output is a matrix, whose first column stores the Banzhaf Interaction index values, and the second column stores the indices of criteria in coalitions.
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.InteractionB(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1), env)
```

fm.InteractionBMob *Banzhaf InteractionB Index computation function in Mobius representation*

Description

Calculates all the Banzhaf InteractionB indices of input criteria for a Mobius fuzzy measure.

Usage

```
fm.InteractionBMob(Mob, env)
```

Arguments

Mob	Fuzzy measure value in Mobius representation
env	Environment variable obtained from fm.Init(n).

Value

output	The output is a matrix, whose first column stores the Banzhaf Interaction index values, and the second column stores the indices of criteria in coalitions.
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.InteractionBMob(c( 0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1), env)
```

fm.InteractionMob *Interaction Index computation function for Mobius fuzzy measure*

Description

Calculates all the interaction indices of input criteria for a Mobius fuzzy measure.

Usage

```
fm.InteractionMob(Mob, env )
```

Arguments

Mob	Fuzzy measure value in Mobius representation
env	Environment variable obtained from fm.Init(n).

Value

output	The output is a matrix, whose first column stores the interaction index values, and the second column stores the indices of criteria in coalitions.
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.InteractionMob(c( 0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1),env)
```

fm.IsMeasureAdditive *IsMeasureAdditive function*

Description

Returns 1 if yes, 0 if no; v is a fuzzy measure in standard representation.

Usage

```
fm.IsMeasureAdditive(v, env)
```

Arguments

v	General fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from fm.Init(n).

Value

output The output is 1 if yes, 0 if no.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureAdditive(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1),env)
```

fm.IsMeasureAdditiveMob

IsMeasureAdditive function in Mobius representation

Description

Returns 1 if yes, 0 if no; v is a fuzzy measure in Mobius representation.

Usage

```
fm.IsMeasureAdditiveMob(Mob, env)
```

Arguments

Mob	Mobius fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from fm.Init(n).

Value

output The output is 1 if yes, 0 if no.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureAdditiveMob(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1),env)
```

`fm.IsMeasureBalanced` *IsMeasureBalanced function*

Description

Returns 1 if yes, 0 if no; v is a fuzzy measure in standard representation.

Usage

```
fm.IsMeasureBalanced(v, env)
```

Arguments

- | | |
|------------------|--|
| <code>v</code> | General fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data. |
| <code>env</code> | Environment variable obtained from <code>fm.Init(n)</code> . |

Value

- | | |
|---------------------|----------------------------------|
| <code>output</code> | The output is 1 if yes, 0 if no. |
|---------------------|----------------------------------|

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureBalanced(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1), env)
```

`fm.IsMeasureBalancedMob`

IsMeasureBalanced function in Mobius representation

Description

Returns 1 if yes, 0 if no; Mob is a fuzzy measure in Mobius representation.

Usage

```
fm.IsMeasureBalancedMob(Mob, env)
```

Arguments

Mob	Mobius fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from fm.Init(n).

Value

output	The output is 1 if yes, 0 if no.
--------	----------------------------------

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureBalancedMob(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1), env)
```

fm.IsMeasureKmaxitive *IsMeasureKmaxitive function*

Description

Returns k; v is a fuzzy measure in standard representation.

Usage

```
fm.IsMeasureKmaxitive(v, env=NULL)
```

Arguments

v	General fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from fm.Init(n).

Value

output	The output is k. If $k=n$ then not k-maxitive
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureKmaxitive(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1), env)
```

fm.IsMeasureKmaxitiveMob

IsMeasureKmaxitive function in Mobius representation

Description

Returns k; mob is a fuzzy measure in Mobius representation.

Usage

```
fm.IsMeasureKmaxitiveMob(Mob, env=NULL)
```

Arguments

- | | |
|-----|---|
| Mob | Mobius fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data. |
| env | Environment variable obtained from fm.Init(n). |

Value

- | | |
|--------|---|
| output | The output is k. If $k=n$ then not k-maxitive |
|--------|---|

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureKmaxitiveMob(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1), env)
```

fm.IsMeasureSelfdual *IsMeasureSelfdual function*

Description

Returns 1 if yes, 0 if no; v is a fuzzy measure in standard representation.

Usage

```
fm.IsMeasureSelfdual(v, env)
```

Arguments

- | | |
|-----|--|
| v | General fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data. |
| env | Environment variable obtained from fm.Init(n). |

Value

output The output is 1 if yes, 0 if no.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureSelfdual(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1),env)
```

fm.IsMeasureSelfdualMob

IsMeasureSelfdual function in Mobius representation

Description

Returns 1 if yes, 0 if no; Mob is a fuzzy measure in Mobius representation.

Usage

```
fm.IsMeasureSelfdualMob(Mob,env)
```

Arguments

Mob	General fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from fm.Init(n).

Value

output The output is 1 if yes, 0 if no.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureSelfdualMob(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1),env)
```

fm.IsMeasureSubadditive

IsMeasureSub additive function

Description

Returns 1 if yes, 0 if no; v is a fuzzy measure in standard representation.

Usage

```
fm.IsMeasureSubadditive(v,env)
```

Arguments

v	General fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from fm.Init(n).

Value

output	The output is 1 if yes, 0 if no.
--------	----------------------------------

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureSubadditive(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1),env)
```

fm.IsMeasureSubadditiveMob

IsMeasureSub additive function in Mobius representation

Description

Returns 1 if yes, 0 if no; v is a fuzzy measure in Mobius representation.

Usage

```
fm.IsMeasureSubadditiveMob(Mob,env)
```

Arguments

Mob	Mobius fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from fm.Init(n).

Value

output	The output is 1 if yes, 0 if no.
--------	----------------------------------

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureSubadditiveMob(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1), env)
```

fm.IsMeasureSubmodular

IsMeasureSub modular function

Description

Returns 1 if yes, 0 if no; v is a fuzzy measure in standard representation.

Usage

```
fm.IsMeasureSubmodular(v, env=NULL)
```

Arguments

v	General fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from fm.Init(n).

Value

output	The output is 1 if yes, 0 if no.
--------	----------------------------------

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureSubmodular(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1), env)
```

fm.IsMeasureSubmodularMob

IsMeasureSubmodular function in Mobius representation

Description

Returns 1 if yes, 0 if no; Mob is a fuzzy measure in Mobius representation.

Usage

```
fm.IsMeasureSubmodularMob(Mob, env=NULL)
```

Arguments

Mob	Mobius fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from fm.Init(n).

Value

output	The output is 1 if yes, 0 if no.
--------	----------------------------------

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureSubmodularMob(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1), env)
```

fm.IsMeasureSuperadditive

IsMeasureSuperadditive function

Description

Returns 1 if yes, 0 if no; v is a fuzzy measure in standard representation.

Usage

```
fm.IsMeasureSuperadditive(v, env=NULL)
```

Arguments

- v General fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
- env Environment variable obtained from fm.Init(n).

Value

- output The output is 1 if yes, 0 if no.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureSuperadditive(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1),env)
```

fm.IsMeasureSuperadditiveMob

IsMeasureSuperadditive function in Mobius representation

Description

Returns 1 if yes, 0 if no; Mob is a fuzzy measure in Mobius representation.

Usage

```
fm.IsMeasureSuperadditiveMob(Mob,env=NULL)
```

Arguments

- Mob Mobius fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
- env Environment variable obtained from fm.Init(n).

Value

- output The output is 1 if yes, 0 if no.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureSuperadditiveMob(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1),env)
```

fm.IsMeasureSupermodular

IsMeasureSupermodular function

Description

Returns 1 if yes, 0 if no; v is a fuzzy measure in standard representation.

Usage

```
fm.IsMeasureSupermodular(v,env=NULL)
```

Arguments

- | | |
|-----|---|
| v | General fuzzy measure of size m=2^n. Its values can be provided by users, or by estimating from empirical data. |
| env | Environment variable obtained from fm.Init(n). |

Value

- | | |
|--------|----------------------------------|
| output | The output is 1 if yes, 0 if no. |
|--------|----------------------------------|

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureSupermodular(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1),env)
```

fm.IsMeasureSupermodularMob

IsMeasureSupermodular function in Mobius representation

Description

Returns 1 if yes, 0 if no; Mob is a fuzzy measure in Mobius representation.

Usage

`fm.IsMeasureSupermodularMob(Mob, env=NULL)`

Arguments

Mob	Mobius fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from <code>fm.Init(n)</code> .

Value

output	The output is 1 if yes, 0 if no.
--------	----------------------------------

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureSupermodularMob(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1),env)
```

fm.IsMeasureSymmetric *IsMeasureSymmetric function*

Description

Returns 1 if yes, 0 if no; v is a fuzzy measure in standard representation.

Usage

`fm.IsMeasureSymmetric(v, env=NULL)`

Arguments

v	General fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from <code>fm.Init(n)</code> .

Value

output The output is 1 if yes, 0 if no.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureSymmetric(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1),env)
```

fm.IsMeasureSymmetricMob

IsMeasureSymmetric function in Mobius representation

Description

Returns 1 if yes, 0 if no; v is a fuzzy measure in Mobius representation.

Usage

```
fm.IsMeasureSymmetricMob(Mob, env=NULL)
```

Arguments

Mob	Mobius fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from fm.Init(n).

Value

output The output is 1 if yes, 0 if no.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.IsMeasureSymmetricMob(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1),env)
```

`fm.is_inset_sparse` *Function for checking if i belongs to the tuple A*

Description

Checks if element i (1-based) belongs to the tuple indexed A (whose cardinality can be 1,2, other (automatically determined)).

Usage

```
fm.is_inset_sparse(A, card, i, envsp=NULL)
```

Arguments

A	Tuple indexed.
card	Whose cardinality can be 1,2, other (automatically determined)
i	Element (1-based)
envsp	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

output	The output is a logical value.
--------	--------------------------------

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
envsp <- fm.PrepareSparseFM(n, vector(), vector())
envsp <- fm.add_singletons_sparse(c(0.2,0.1,0.2),envsp)
envsp <- fm.add_tuple_sparse(c(1,2,3),0.4,envsp);

fm.is_inset_sparse(0,3,1,envsp)
fm.is_inset_sparse(0,3,4,envsp)

envsp <- fm.FreeSparseFM(envsp)
```

fm.is_subset_sparse *Function for checking if tuple B is subset of tuple A*

Description

Checks if tuple B is a subset of tuple A, The cardinalities of both tuples need to be supplied.

Usage

```
fm.is_subset_sparse(A, cardA, B, cardB, envsp = NULL)
```

Arguments

A	Tuple
cardA	Whose cardinality can be 1,2, other (automatically determined)
B	Tuple, tup=0
cardB	Whose cardinality can be 1,2, other (automatically determined)
envsp	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

output The output is a logical value.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
envsp <- fm.PrepareSparseFM(n, vector(), vector())
envsp <- fm.add_singletons_sparse(c(0.2,0.1,0.2),envsp)
envsp <- fm.add_tuple_sparse(c(1,2,3),0.4,envsp);
envsp <- fm.add_pair_sparse(1,2,0.2,envsp);
envsp <- fm.add_pair_sparse(1,3,0.3,envsp);

fm.is_subset_sparse(0,3,0,2,envsp) #is 0th pair a subset of the 0th tuple?
fm.is_subset_sparse(0,3,1,2,envsp) #is 1th pair a subset of the 0th tuple?

envsp<-fm.FreeSparseFM(envsp)
```

fm.max_subset_sparse *Maximun of x computation function in sparse representation*

Description

Calculates maximum of x with the indices belonging to tuple indexed as S

Usage

```
fm.max_subset_sparse(x, S, cardS, envsp=NULL)
```

Arguments

x	Input vector of size n, containing utility value of input criteria. x is in [0,1].
S	Indices belonging to tuple indexed
cardS	Cardinality cardS
envsp	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

output	The output is the maximum of x with the indices belonging to tuple indexed as S
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
envsp <- fm.PrepareSparseFM(n, vector(), vector())
envsp <- fm.add_singletons_sparse(c(0.2,0.1,0.2),envsp)
envsp <- fm.add_tuple_sparse(c(1,2,3),0.4,envsp);

fm.max_subset_sparse(c(0.1,0.05,0.2),0,3,envsp)
envsp <- fm.FreeSparseFM(envsp)
```

fm.min_subset_sparse *Minimun of x computation function in sparse representation*

Description

Calculates minimum of x with the indices belonging to tuple indexed as S

Usage

```
fm.min_subset_sparse(x, S, cardS, envsp=NULL)
```

Arguments

x	Input vector of size n, containing utility value of input criteria. x is in [0,1].
S	Indices belonging to tuple indexed
cardS	Cardinality cardS
envsp	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

output	The output is the minimum of x with the indices belonging to tuple indexed as S
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
envsp <- fm.PrepareSparseFM(n, vector(), vector())
envsp <- fm.add_singletons_sparse(c(0.2,0.1,0.2),envsp)
envsp <- fm.add_tuple_sparse(c(1,2,3),0.4,envsp);

fm.min_subset_sparse(c(0.1,0.05,0.2),0,3,envsp)
envsp <- fm.FreeSparseFM(envsp)
```

fm.Mobius*Mobius transform function*

Description

Calculates Mobius representation of general fuzzy measure, the input and output is an array of size $2^n=m$ in binary ordering.

Usage

```
fm.Mobius(v, env=NULL)
```

Arguments

- | | |
|-----|---|
| v | Fuzzy measure value in standard representation. |
| env | Environment variable obtained from fm.Init(n). |

Value

- | | |
|--------|---|
| output | The output is the fuzzy measure in Mobius representation. |
|--------|---|

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.Mobius(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1), env)
```

fm.NonadditivityIndex *Nonadditivity index computation function*

Description

Calculate the nonadditivity indices of input criteria from general fuzzy measure.

Usage

```
fm.NonadditivityIndex(v, env=NULL)
```

Arguments

- | | |
|-----|--|
| v | Fuzzy measure in general representation. |
| env | Environment variable obtained from fm.Init(n). |

Value

- output** The output is an array of size 2^n , which contain nonadditivity indices of input criteria coalitions.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.NonadditivityIndex(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1), env)
```

fm.NonadditivityIndexMob

Nonadditivity index computation function in Mobius representation

Description

Calculate the nonadditivity indices of input criteria from general fuzzy measure in Mobius representation.

Usage

```
fm.NonadditivityIndexMob(Mob, env=NULL)
```

Arguments

- Mob** Fuzzy measure in Mobius representation.
env Environment variable obtained from fm.Init(n).

Value

- output** The output is an array of size 2^n , which contain nonadditivity indices of input criteria coalitions.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
NonadditivityIndex <- fm.NonadditivityIndexMob(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1), env)
```

fm.NonmodularityIndex *Nonmodularity index computation function*

Description

Calculate all the $m = 2^n$ nonmodularity indices of fuzzy measure v given in standard representation

Usage

```
fm.NonmodularityIndex(v, env = NULL)
```

Arguments

- | | |
|-----|--|
| v | Fuzzy measure in general representation. |
| env | Environment variable obtained from fm.Init(n). |

Value

- | | |
|--------|------------------------------------|
| output | The output is an array of size m |
|--------|------------------------------------|

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
Nonmodularityindex <- fm.NonmodularityIndex(c(0,0.3,0.5,0.6,0.4,0.8,0.7,1),env)
```

fm.NonmodularityIndexKinteractive
NonmodularityIndexKinteractive computation function

Description

Calculate all the $m = 2^n$ nonmodularity indices of k -interactive fuzzy measure v given in standard representation (in cardinality ordering)

Usage

```
fm.NonmodularityIndexKinteractive(v, env = NULL, kadd = "NA")
```

Arguments

v	Fuzzy measure in general representation.
env	Environment variable obtained from fm.Init(n).
kadd	kadd is the value of k-additivity, which is used for reducing the complexity of fuzzy measures. default value is kadd = n. $1 < \text{kadd} < n+1$; if kdd=n - f.m. is unrestricted

Value

output	The output is an array of size m.
--------	-----------------------------------

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.NonmodularityIndexKinteractive(c(0,0.3,0.5,0.6,0.4,0.8,0.7,1),env,2)
```

fm.NonmodularityIndexMob

Nonmodularityindex computation function in Mobius representation

Description

Calculates all the nonmodularity indices of fuzzy measure in Mobius representation representation

Usage

```
fm.NonmodularityIndexMob(Mob, env = NULL)
```

Arguments

Mob	Fuzzy measure in Mobius representation of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from fm.Init(n).

Value

output	The output is an array of size m
--------	----------------------------------

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.NonmodularityIndexMob(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1), env)
```

fm.NonmodularityIndexMobkadditive

Function for calculating all Nonmodularity indices of k-additive in Mobius representation

Description

Calculate all the $m = 2^n$ nonmodularity indices of k-additive in Mobius representation(in cardinality ordering)

Usage

```
fm.NonmodularityIndexMobkadditive(Mob, env = NULL, kadd = "NA")
```

Arguments

Mob	Fuzzy measure in Mobius representation of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data
env	Environment variable obtained from fm.Init(n).
kadd	kadd is the value of k-additivity, which is used for reducing the complexity of fuzzy measures. default value is kadd = n. $1 < \text{kadd} < n+1$; if kdd=n - f.m. is unrestricted.

Value

output	The output is an array of size m.
--------	-----------------------------------

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.NonmodularityIndexMobkadditive(c(0.0,0.3,0.5,-0.2,0.4,0.1,-0.2,0.1),env,2)
```

fm.NonmodularityIndex_sparse*Nonmodularity index computation function in sparse representation***Description**

Calculate all 2^n nonmodularity indices using Mobius transform of a fuzzy measure of lenght $2^n=m$, using sparse representation

Usage

```
fm.NonmodularityIndex_sparse( n, envsp=NULL)
```

Arguments

n	Number of inputs
envsp	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

output	The output is all 2^n nonmodularity indice.
---------------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
tups<-vector()
tupsidx<-vector()
envsp <- fm.PrepareSparseFM(n, tups,tupsidx)
envsp <- fm.add_singletons_sparse(c(0.2,0.1,0.2),envsp)
envsp <- fm.add_pair_sparse(1,2,0.4,envsp);

fm.NonmodularityIndex_sparse(3,envsp)
envsp <- fm.FreeSparseFM(envsp)
```

fm.OrnessChoquet

*OrnessChoquet function***Description**

Calculate Orness value of the Choquet integral of the fuzzy measure, where v is a standard representation.

Usage

```
fm.OrnessChoquet(v, env=NULL)
```

Arguments

v	Standard fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from fm.Init(n).

Value

output	The output is the Orness the Choquet integral for the fuzzy measure.
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.OrnessChoquet(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1), env)
```

fm.OrnessChoquetMob

*OrnessChoquet function in Mobius representation***Description**

Calculate Orness value of the Choquet integral of the fuzzy measure, where Mob is the Mobius representation.

Usage

```
fm.OrnessChoquetMob(Mob, env=NULL)
```

Arguments

Mob	Mobius fuzzy measure of size $m=2^n$. Its values can be provided by users, or by estimating from empirical data.
env	Environment variable obtained from fm.Init(n).

Value

output	The output is the Orness the Choquet integral for the fuzzy measure.
--------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.OrnessChoquetMob(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1), env)
```

fm.populate_fm_2add_sparse

Function for populating 2-additive sparse capacity

Description

Populate 2-additive sparse capacity with nonzero values using the singletons and two arrays of indices (of size numpairs).

Usage

```
fm.populate_fm_2add_sparse(singletons, numpairs, pairs, indicesp1, indicesp2, envsp)
```

Arguments

singletons	Singletons 0-based.
numpairs	Size numpairs.
pairs	Array 0-based.
indicesp1	Array of indices of Size numpairs.need to be 1-based.
indicesp2	Array of indices of Size numpairs.need to be 1-based.
envsp	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

output	The output is Populate 2-additive sparse capacity with nonzero values using the singletons and two arrays of indices (of size numpairs)
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
envsp <- fm.PrepareSparseFM(n, vector(), vector())

envsp <- fm.populate_fm_2add_sparse(c(0.1,0.2,0.3), 3,
                                      c(0.4,0.5,0.6), c(1,1,2), c(2,3,3), envsp)
envsp
envsp <- fm.FreeSparseFM(envsp)
```

fm.populate_fm_2add_sparse_from2add

Function for populating 2-additive sparse capacity from 2-additive capacity

Description

Given 2-additive capacity singletons+pairs in one array v , selects nonzero pairs and populates sparse capacity envsp

Usage

```
fm.populate_fm_2add_sparse_from2add(n, v, envsp=NULL)
```

Arguments

n	Number of inputs
v	Pairs in one array v
envsp	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

output	The output is a nonzero pairs and populates sparse capacity envsp
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
envsp <- fm.PrepareSparseFM(n, vector(), vector())
envsp <- fm.populate_fm_2add_sparse_from2add(3,c(0.4,0.5,0.6, 0, 0, 0.1),envsp)
envsp
envsp <- fm.FreeSparseFM(envsp)
```

fm.PrepareSparseFM *PrepareSparseFM preparation function*

Description

This function initialises Sparse representation structure. It is used to allocate storage and later populate these values

Usage

```
fm.PrepareSparseFM(n, tups, tupsidx)
```

Arguments

<code>n</code>	Number of inputs
<code>tups</code>	Tuples to be added (can be null vector)
<code>tupsidx</code>	Cardinalities and indices (1-based) of the elements of tuples (can be null vector)

Value

`output` The output allocate storage and later populate these values. `envsp`

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n<-3
envsp <- fm.PrepareSparseFM(n, vector(), vector())
envsp <- fm.FreeSparseFM(envsp)
envsp <- fm.PrepareSparseFM(n, c(0.2,0.4,0.1), c(2,1,2,2,1,3,3,1,2,3))
envsp
envsp <- fm.FreeSparseFM(envsp)
```

fm.Shapley*Shapley value computation function***Description**

Calculates the Shapley values of input criteria from general fuzzy measure,

Usage

```
fm.Shapley(v, env=NULL)
```

Arguments

- | | |
|-----|--|
| v | Fuzzy measure in general representation. |
| env | Environment variable obtained from fm.Init(n). |

Value

- | | |
|--------|---|
| output | The output is an array of size n, which contain Shapley values of input criteria. |
|--------|---|

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.Shapley(c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1), env)
```

fm.Shapley2addMob*Function for calculating Shapley values of 2-additive fuzzy measure in Mobius representation***Description**

Calculate the Shapley values of a 2-additive fuzzy measure for n inputs given in Mobius representation. The results are in arrays.

Usage

```
fm.Shapley2addMob(n, Mob)
```

Arguments

- | | |
|-----|--|
| n | Number of inputs |
| Mob | Fuzzy measure value in Mobius representation |

Value

output The output is an array of size n, which contain Shapley indices of input criteria.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
Shapley <- fm.Shapley2addMob(3, c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1))
```

fm.ShapleyMob

Shapley value computation function in Mobius representation

Description

Calculate the Shapley indices of input criteria from general fuzzy measure in Mobius representation.

Usage

```
fm.ShapleyMob(Mob, env=NULL)
```

Arguments

Mob	Fuzzy measure in Mobius representation.
env	Environment variable obtained from fm.Init(n).

Value

output The output is an array of size n, which contain Shapley values of input criteria.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.ShapleyMob(c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1), env)
```

`fm.ShapleyMob_sparse` *Shapley values computation function in sparse representation*

Description

Calculate Shapley values vectors of size n of a sparse fuzzy measure

Usage

```
fm.ShapleyMob_sparse(n, envsp=NULL)
```

Arguments

<code>n</code>	Size of values vectors
<code>envsp</code>	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

`output` The output is Shapley values vectors of size n of a sparse fuzzy measure.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
envsp <- fm.PrepareSparseFM(n, c(0.2,0.4,0.1), c(2,1,2,2,1,3,3,1,2,3))

fm.ShapleyMob_sparse(3, envsp)

envsp <- fm.FreeSparseFM(envsp)
```

`fm.ShowCoalitions` *Show Coalitions function*

Description

Return the decimal expression for the subsets A. In binary and in cardinality ordering respectively.

Usage

```
fm.ShowCoalitions(env = NULL)
```

Arguments

env Environment variable obtained from fm.Init(n).

Value

output is the array of integers which show the decimal expressions for all 2^n coalitions.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
ShowCoalitions <- fm.ShowCoalitions(env)
ShowCoalitions
```

fm.ShowCoalitionsCard Show CoalitionsCard function**Description**

Return the decimal expression for the subsets A. In binary and in cardinality ordering respectively.

Usage

```
fm.ShowCoalitionsCard(env = NULL)
```

Arguments

env Environment variable obtained from fm.Init(n).

Value

output The output the decimal expression for the subsets A. It is the array of integers containing the decimal expressions for all 2^n coalitions.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
CoalitionsCard <- fm.ShowCoalitionsCard(env)
CoalitionsCard
```

`fm.sparse_get_pairs` *Get pairs computation function in sparse representation*

Description

Export the internal arrays of the sparse capacity as arrays of singletons, pairs and tuples.

Usage

```
fm.sparse_get_pairs( envsp=NULL)
```

Arguments

<code>envsp</code>	Structure required for sparse representation which stores the relevant values (k -tuples). It is obtained from <code>fm.PrepareSparseFM(n)</code> .
--------------------	--

Value

<code>output</code>	The output is the array of pairs and their number.
---------------------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
envsp <- fm.PrepareSparseFM(n)
envsp <- fm.add_pair_sparse(1,2, 0.4, envsp)
envsp <- fm.add_pair_sparse(1,3, 0.3, envsp)

pairs <- fm.sparse_get_pairs(envsp)
pairs
envsp <- fm.FreeSparseFM(envsp)
```

`fm.sparse_get_singletons`

Get singletons of sparse fuzzy measure

Description

Export the internal arrays of the sparse capacity as arrays of singletons, pairs and tuples.

Usage

```
fm.sparse_get_singletons( envsp=NULL)
```

Arguments

envsp Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

output The output is the numbers of pairs and tuples.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
envsp <- fm.PrepareSparseFM(n)
envsp <- fm.add_singletons_sparse(c(0, 0.3, 0.5), envsp)
singletons <- fm.sparse_get_singletons(envsp)
singletons
envsp <- fm.FreeSparseFM(envsp)
```

fm.sparse_get_tuples *Get tuples of a sparse fuzzy measure*

Description

Export the internal arrays of the sparse capacity as arrays of singletons, pairs and tuples.

Usage

```
fm.sparse_get_tuples(envsp=NULL)
```

Arguments

envsp Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

output The output is the numbers of pairs and tuples.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
envsp <- fm.PrepareSparseFM(n)
envsp <- fm.add_tuple_sparse(c(1,2,3),0.2,envsp)
envsp <- fm.add_tuple_sparse(c(1,3,4),0.3,envsp)

tuples <- fm.sparse_get_tuples(envsp)
tuples
envsp <- fm.FreeSparseFM(envsp)
```

fm.Sugeno

Sugeno computation function

Description

Calculate the value of a Sugeno integral of input x, with fuzzy measure in standard representation

Usage

```
fm.Sugeno(x, v, env=NULL)
```

Arguments

- x Input vector of size n, containing utility value of input criteria. x is in [0,1].
- v General fuzzy measure of size m=2^n. Its values can be provided by users, or by estimating from empirical data.
- env Environment variable obtained from fm.Init(n).

Value

- output The output is a single value of the computed Sugeno integral.

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.Sugeno(c(0.6, 0.3, 0.8), c(0, 0.3, 0.5, 0.6, 0.4, 0.8, 0.7, 1),env)
```

fm.SugenoMob*Sugeno function in Mobius representation***Description**

Calculate the value of a Sugeno integral of input x, with fuzzy measure in mobius representation

Usage

```
fm.SugenoMob(x, Mob, env=NULL)
```

Arguments

- | | |
|-----|--|
| x | Input vector of size n, containing utility value of input criteria. x is in [0,1]. |
| Mob | Mobius fuzzy measure of size m=2^n. Its values can be provided by users, or by estimating from empirical data. |
| env | Environment variable obtained from fm.Init(n). |

Value

- | | |
|--------|---|
| output | The output is a single value of the computed Sugeno integral. |
|--------|---|

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.SugenoMob(c(0.6, 0.3, 0.8), c(0.0, 0.3, 0.5, -0.2, 0.4, 0.1, -0.2, 0.1), env)
```

fm.test*Test function***Description**

This function provide some examples of how fuzzy measure operation in this toolbox are used. It can be used to test if the toolbox has been installed successfully or not.

Usage

```
fm.test()
```

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
fm.test()
```

fm.tuple_cardinality_sparse
Tuple cardinality

Description

Returns the cardinality of the tuple numbered i in the list of tuples.

Usage

```
fm.tuple_cardinality_sparse(i, envsp = NULL)
```

Arguments

i	In the list of tuples.
envsp	Structure required for sparse representation which stores the relevant values (k-tuples). It is obtained from fm.PrepareSparseFM(n).

Value

output	The output is the cardinality of the tuple numbered i in the list of tuple.
--------	---

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
n <- 3
envsp <- fm.PrepareSparseFM(n, c(0.2,0.4,0.1), c(2,1,2,2,1,3,3,1,2,3))
fm.tuple_cardinality_sparse(0,envsp)
envsp <- fm.FreeSparseFM(envsp)
```

<code>fm.Zeta</code>	<i>Zeta transform function</i>
----------------------	--------------------------------

Description

Calculate the general fuzzy measure from Mobius representation. The input and output is an array of size $2^n=m$ in binary ordering. This is the inverse of the Mobius function.

Usage

```
fm.Zeta(Mob,env)
```

Arguments

<code>Mob</code>	Fuzzy measure value in Mobius representation.
<code>env</code>	Environment variable obtained from fm.Init(n).

Value

<code>output</code>	The output is the fuzzy measure in general representation.
---------------------	--

Author(s)

Gleb Beliakov, Andrei Kelarev, Quan Vu, Daniela L. Calderon, Deakin University

Examples

```
env<-fm.Init(3)
fm.Zeta(c(0.0,0.3,0.5,-0.2,0.4,0.1,-0.2,0.1),env)
```

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