Package ‘PerRegMod’

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Type Package

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Title Fitting Periodic Coefficients Linear Regression Models

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Description Provides tools for fitting periodic coefficients regression models to data where periodicity plays a crucial role. It allows users to model and analyze relationships between variables that exhibit cyclical or seasonal patterns, offering functions for estimating parameters and testing the periodicity of coefficients in linear regression models. For simple periodic coefficient regression model see Regui et al. (2024) <doi:10.1080/03610918.2024.2314662>.

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Contents

`A_x_B` *A Kronecker product B*

Description

A_x_B() function gives A Kronecker product B

Usage

```
A_x_B(A,B)
```

Arguments

`A`	A matrix.
`B`	A matrix.

Value

`A_x_B(A, B)`	returns the matrix A Kronecker product B, $A \otimes B$

Examples

```
A=matrix(rep(1,6),3,2)
B=matrix(seq(1,8),2,4 )
A_x_B(A,B)
```

`check_periodicity` *Checking the periodicity of parameters in the regression model*

Description

check_periodicity() function allows to detect the periodicity of parameters in the regression model using pseudo_gaussian_test. See *Regui et al. (2024)* for periodic simple regression model. $T^{(n)} = \left(\mathbf{\Delta}_1^{\circ(n)'}, \mathbf{\Delta}_2^{\circ(n)'}, \mathbf{\Delta}_3^{\circ(n)'}\right) \begin{pmatrix} \mathbf{\Gamma}_1^{\circ} & \mathbf{\Gamma}_{12}^{\circ} & \mathbf{0} \\ \mathbf{\Gamma}_{12}^{\circ} & \mathbf{\Gamma}_{22}^{\circ} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Gamma}_{33}^{\circ} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{\Delta}_1^{\circ(n)} \\ \mathbf{\Delta}_2^{\circ(n)} \\ \mathbf{\Delta}_3^{\circ(n)} \end{pmatrix}$, where $\mathbf{\Delta}_1^{\circ(n)} = n^{\frac{-1}{2}} \sum_{r=0}^{m-1} \begin{pmatrix} \widehat{\phi}(Z_{1+Sr}) - \widehat{\phi}(Z_{S+Sr}) \\ \vdots \\ \widehat{\phi}(Z_{S-1+Sr}) - \widehat{\phi}(Z_{S+Sr}) \end{pmatrix}$

$\mathbf{\Delta}_2^{\circ(n)} = \frac{n^{\frac{-1}{2}}}{2\widehat{\sigma}} \sum_{r=0}^{m-1} \begin{pmatrix} \widehat{\psi}(Z_{1+Sr}) - \widehat{\psi}(Z_{S+Sr}) \\ \vdots \\ \widehat{\psi}(Z_{S-1+Sr}) - \widehat{\psi}(Z_{S+Sr}) \end{pmatrix}$,

$$\boldsymbol{\Delta}_3^{\circ(n)} = n^{\frac{-1}{2}} \sum_{r=0}^{m-1} \begin{pmatrix} \widehat{\phi}(Z_{1+Sr})\mathbf{K}_1^{(n)}\mathbf{X}_{1+Sr} - \widehat{\phi}(Z_{S+Sr})\mathbf{K}_S^{(n)}\mathbf{X}_{S+Sr} \\ \vdots \\ \widehat{\phi}(Z_{S-1+Sr})\mathbf{K}_{S-1}^{(n)}\mathbf{X}_{S-1+Sr} - \widehat{\phi}(Z_{S+Sr})\mathbf{K}_S^{(n)}\mathbf{X}_{S+Sr} \end{pmatrix}, \ \boldsymbol{\Gamma}_{11}^{\circ} = \frac{\widehat{I}_n}{S}\Sigma,$$

$\boldsymbol{\Gamma}_{22}^{\circ} = \frac{\widehat{I}_n}{4S\widehat{\sigma}^2}\Sigma$, $\boldsymbol{\Gamma}_{12}^{\circ} = \frac{\widehat{N}_n}{2S\widehat{\sigma}}\Sigma$, and $\boldsymbol{\Gamma}_{33}^{\circ} = \frac{\widehat{I}_n}{S}\Sigma \otimes \mathbf{I}_{p\times p}$ with $\widehat{I}_n = \frac{1}{nT}\sum_{s=1}^{S}\sum_{r=0}^{m-1}\widehat{\phi}^2\left(\frac{\widehat{Z}_{s+Sr}}{\widehat{\sigma}_s}\right)$,

$\widehat{N}_n = \frac{1}{nT}\sum_{s=1}^{S}\sum_{r=0}^{m-1}\widehat{\phi}^2\left(\frac{\widehat{Z}_{s+Sr}}{\widehat{\sigma}_s}\right)\frac{\widehat{Z}_{s+Sr}}{\widehat{\sigma}_s}$,

$$\Sigma = \begin{bmatrix} 2 & 1 & \dots & 1 \\ 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 1 & \dots & 1 & 2 \end{bmatrix}, \ Z_{s+Sr} = \frac{y_{s+Sr} - \widehat{\mu}_s - \sum_{j=1}^{p}\widehat{\beta}_s^j x_{s+Sr}^j}{\widehat{\sigma}_s}, \ \mathbf{X}_{s+Sr} = \left(x_{s+Sr}^1, \dots, x_{s+Sr}^p\right)',$$

$$\mathbf{K}_s^{(n)} = \begin{bmatrix} \overline{(x_s^1)^2} & & \overline{x_s^i x_s^j} \\ & \ddots & \\ \overline{x_s^j x_s^i} & & \overline{(x_s^p)^2} \end{bmatrix}^{\frac{-1}{2}},$$

$\overline{x_s^i x_s^j} = \frac{1}{m}\sum_{r=0}^{m-1} x_{s+Sr}^i x_{s+Sr}^j$, $\overline{(x_s^i)^2} = \frac{1}{m}\sum_{r=0}^{m-1}(x_{s+Sr}^i)^2$, $\widehat{\psi}(x) = x\widehat{\phi}(x) - 1$, and

$$\widehat{\phi}(x) = \frac{1}{b_n^2}\frac{\sum_{s=1}^{S}\sum_{r=0}^{m-1}(x - Z_{s+Sr})\exp\left(-\frac{(x-Z_{s+Sr})^2}{2b_n^2}\right)}{\sum_{s=1}^{S}\sum_{r=0}^{m-1}\exp\left(-\frac{(x-Z_{s+Sr})^2}{2b_n^2}\right)} \text{ with } b_n \to 0.$$

Usage

```
check_periodicity(x,y,s)
```

Arguments

x	A list of independent variables with dimension p.
y	A response variable.
s	A period of the regression model.

Value

`check_periodicity()`
returns the value of observed statistic, $T^{(n)}$, degrees of freedom, $(S-1)\times(p+2)$, and p-value

References

Regui, S., Akharif, A., & Mellouk, A. (2024). "Locally optimal tests against periodic linear regression in short panels." Communications in Statistics-Simulation and Computation, 1–15. doi:10.1080/03610918.2024.2314662

Examples

```
library(expm)
set.seed(6)
n=400
s=4
x1=rnorm(n,0,1.5)
x2=rnorm(n,0,0.9)
x3=rnorm(n,0,2)
x4=rnorm(n,0,1.9)
y=rnorm(n,0,2.5)
x=list(x1,x2,x3,x4)
check_periodicity(x,y,s)
```

DELTA *Calculating the component of vector DELTA*

Description

DELTA() function gives the value of the component of vector DELTA $\mathbf{\Delta}$. See *Regui et al. (2024)* for periodic simple regression model. $\mathbf{\Delta} = \begin{bmatrix} \mathbf{\Delta}_1 \\ \mathbf{\Delta}_2 \\ \mathbf{\Delta}_3 \end{bmatrix}$, where $\mathbf{\Delta}_1$ is a vector of dimension S with component $\frac{n^{\frac{-1}{2}}}{\widehat{\sigma}_s} \sum_{r=0}^{m-1} \widehat{\phi}(Z_{s+Sr,t})$, $\mathbf{\Delta}_2$ is a vector of dimension pS with component $\frac{n^{\frac{-1}{2}}}{\widehat{\sigma}_s} \sum_{r=0}^{m-1} \widehat{\phi}(Z_{s+Sr}) K_s^{(n)} \mathbf{X}_{s+Sr}$, $\mathbf{\Delta}_3$ is a vector of dimension S with component $\frac{n^{\frac{-1}{2}}}{2\widehat{\sigma}_s^2} \sum_{r=0}^{m-1} Z_{s+Sr} \widehat{\phi}(Z_{s+Sr}) - 1$.

Usage

```
DELTA(x,phi,s,e,sigma)
```

Arguments

x	A list of independent variables with dimension p.
phi	phi_n.
s	A period of the regression model.
e	The residuals vector.
sigma	sd_estimation_for_each_s.

Value

DELTA()	returns the values of $\mathbf{\Delta}$. See *Regui et al. (2024)* for simple periodic coefficients regression model.

References

Regui, S., Akharif, A., & Mellouk, A. (2024). "Locally optimal tests against periodic linear regression in short panels." Communications in Statistics-Simulation and Computation, 1–15. doi:10.1080/03610918.2024.2314662

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estimate_para_adaptive_method

Adaptive estimator for periodic coefficients regression model

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Description

estimate_para_adaptive_method() function gives the adaptive estimation of parameters of a periodic coefficients regression model.

Usage

```
estimate_para_adaptive_method(n,s,y,x)
```

Arguments

n	The length of vector y.
s	A period of the regression model.
y	A response variable.
x	A list of independent variables with dimension p.

Value

beta_ad	Parameters to be estimated.

Examples

```
set.seed(6)
n=400
s=4
x1=rnorm(n,0,1.5)
x2=rnorm(n,0,0.9)
x3=rnorm(n,0,2)
x4=rnorm(n,0,1.9)
y=rnorm(n,0,2.5)
x=list(x1,x2,x3,x4)
model=lm(y~x1+x2+x3+x4)
z=model$residuals
estimate_para_adaptive_method(n,s,y,x)
```

GAMMA *Calculating the component of matrix GAMMA*

Description

GAMMA() function gives the value of the component of matrix GAMMA $\mathbf{\Gamma}$. See *Regui et al. (2024)* for periodic simple regression model. $\mathbf{\Gamma} = \frac{1}{S}\begin{bmatrix} (\mathbf{\Gamma}_{11})_{S\times S} & \mathbf{0} & \mathbf{\Gamma}_{13} \\ \mathbf{0} & (\mathbf{\Gamma}_{22})_{pS\times pS} & \mathbf{0} \\ \mathbf{\Gamma}_{13} & \mathbf{0} & (\mathbf{\Gamma}_{33})_{S\times S} \end{bmatrix}$, where $\mathbf{\Gamma}_{11} = \widehat{I}_n \mathrm{diag}(\frac{1}{\widehat{\sigma}_1^2}, ..., \frac{1}{\widehat{\sigma}_S^2})$, $\mathbf{\Gamma}_{13} = \frac{\widehat{N}_n}{2}\mathrm{diag}(\frac{1}{\widehat{\sigma}_1^3}, ..., \frac{1}{\widehat{\sigma}_S^3})$, $\mathbf{\Gamma}_{22} = \widehat{I}_n \mathrm{diag}(\frac{1}{\widehat{\sigma}_1^2}, ..., \frac{1}{\widehat{\sigma}_S^2}) \otimes \mathbf{I}_p$, $\mathbf{\Gamma}_{33} = \frac{\widehat{J}_n}{4}\mathrm{diag}(\frac{1}{\widehat{\sigma}_1^4}, ..., \frac{1}{\widehat{\sigma}_S^4})$, $\widehat{I}_n = \frac{1}{nT}\sum_{s=1}^{S}\sum_{r=0}^{m-1}\widehat{\phi}^2\left(\frac{\widehat{Z}_{s+Sr}}{\widehat{\sigma}_s}\right)$, $\widehat{N}_n = \frac{1}{nT}\sum_{s=1}^{S}\sum_{r=0}^{m-1}\widehat{\phi}^2\left(\frac{\widehat{Z}_{s+Sr}}{\widehat{\sigma}_s}\right)\frac{\widehat{Z}_{s+Sr}}{\widehat{\sigma}_s}$, $\widehat{J}_n = \frac{1}{nT}\sum_{s=1}^{S}\sum_{r=0}^{m-1}\widehat{\phi}^2\left(\frac{\widehat{Z}_{s+Sr}}{\widehat{\sigma}_s}\right)\left(\frac{\widehat{Z}_{s+Sr}}{\widehat{\sigma}_s}\right)^2 - 1$, and

$$\widehat{\phi}(x) = \frac{1}{b_n^2}\frac{\sum_{s=1}^{S}\sum_{r=0}^{m-1}(x - Z_{s+Sr})\exp\left(-\frac{(x-Z_{s+Sr})^2}{2b_n^2}\right)}{\sum_{s=1}^{S}\sum_{r=0}^{m-1}\exp\left(-\frac{(x-Z_{s+Sr})^2}{2b_n^2}\right)}$$ with $b_n \to 0$.

Usage

```
GAMMA(x,phi,s,z,sigma)
```

Arguments

x	A list of independent variables with dimension p.
phi	phi_n.
s	A period of the regression model.
z	The residuals vector.
sigma	sd_estimation_for_each_s.

Value

GAMMA()	returns the matrix $\mathbf{\Gamma}$. See *Regui et al. (2024)* for simple periodic coefficients regression model.

References

Regui, S., Akharif, A., & Mellouk, A. (2024). "Locally optimal tests against periodic linear regression in short panels." Communications in Statistics-Simulation and Computation, 1–15. doi:10.1080/03610918.2024.2314662

lm_per *Fitting periodic coefficients regression model by using LSE*

Description

lm_per() function gives the least squares estimation of parameters, intercept μ_s, slope $\boldsymbol{\beta}_s$, and standard deviation σ_s, of a periodic coefficients regression model using LSE_Reg_per and sd_estimation_for_each_s functions. $\widehat{\boldsymbol{\vartheta}} = \left(X'X\right)^{-1} X'Y$ where $X = \begin{bmatrix} & \mathbf{X}_1^1 & 0 & \dots & 0 & & \mathbf{X}_1^p & 0 & \dots & 0 \\ & 0 & \mathbf{X}_2^1 & \dots & 0 & & 0 & \mathbf{X}_2^p & \dots & 0 \\ \mathbf{I}_S \otimes \mathbf{1}_m & 0 & 0 & \ddots & \vdots & \dots & 0 & 0 & \ddots & \vdots \\ & 0 & 0 & 0 & \mathbf{X}_S^1 & & 0 & 0 & 0 & \mathbf{X}_S^p \end{bmatrix}$,

$\mathbf{X}_s^j = \left(x_s^j, ..., x_{s+(m-1)S}^j\right)'$, $Y = (\mathbf{Y}_1', ..., \mathbf{Y}_S')'$, $\mathbf{Y}_s = (y_s, ..., y_{(m-1)S+s})'$, $\epsilon = (\epsilon_1', ..., \epsilon_S')'$, $\epsilon_s = (\varepsilon_s, ..., \varepsilon_{(m-1)S+s})'$, $\mathbf{1}_m$ is a vector of ones of dimension m, $\mathbf{I}_S$ is the identity matrix of dimension S, $\otimes$ denotes the Kronecker product, and $\boldsymbol{\vartheta} = \left(\boldsymbol{\mu}', \boldsymbol{\beta}'\right)'$ with $\boldsymbol{\mu} = (\mu_1, ..., \mu_S)'$ and $\boldsymbol{\beta} = (\beta_1^1, ..., \beta_S^1; ...; \beta_1^p, ..., \beta_S^p)'$.

Usage

```
lm_per(x,y,s)
```

Arguments

x	A list of independent variables with dimension p.
y	A response variable.
s	A period of the regression model.

Value

Residuals	the residuals, that is response minus fitted values
Coefficients	a named vector of coefficients
Root mean square error	The root mean square error

Examples

```
set.seed(6)
n=400
s=4
x1=rnorm(n,0,1.5)
x2=rnorm(n,0,0.9)
x3=rnorm(n,0,2)
x4=rnorm(n,0,1.9)
y=rnorm(n,0,2.5)
x=list(x1,x2,x3,x4)
lm_per(x,y,s)
```

`lm_per_AE` *Fitting periodic coefficients regression model by using Adaptive estimation method*

Description

lm_per_AE() function gives the adaptive estimation of parameters, intercept μ_s, slope $\boldsymbol{\beta}_s$, and standard deviation σ_s, of a periodic coefficients regression model. $\widehat{\boldsymbol{\theta}}_{AE} = \widehat{\boldsymbol{\vartheta}}_{LSE} + \frac{1}{\sqrt{n}}\boldsymbol{\Gamma}^{-1}\boldsymbol{\Delta}$.

Usage

```
lm_per_AE(x,y,s)
```

Arguments

- `x` A list of independent variables with dimension p.
- `y` A response variable.
- `s` A period of the regression model.

Value

- `Residuals` the residuals, that is response minus fitted values
- `Coefficients` a named vector of coefficients
- `Root mean square error` The root mean square error

Examples

```
set.seed(6)
n=200
s=2
x1=rnorm(n,0,1.5)
x2=rnorm(n,0,0.9)
x3=rnorm(n,0,2)
x4=rnorm(n,0,1.9)
y=rnorm(n,0,2.5)
x=list(x1,x2,x3,x4)
lm_per_AE(x,y,s)
```

`LSE_Reg_per` *Least squares estimator for periodic coefficients regression model*

Description

LSE_Reg_per() function gives the least squares estimation of parameters of a periodic coefficients regression model.

Usage

```
LSE_Reg_per(x,y,s)
```

Arguments

`x`	A list of independent variables with dimension p.
`y`	A response variable.
`s`	A period of the regression model.

Value

`beta`	Parameters to be estimated.
`X`	Matrix of predictors.
`Y`	The response vector.

Examples

```
set.seed(6)
n=400
s=4
x1=rnorm(n,0,1.5)
x2=rnorm(n,0,0.9)
x3=rnorm(n,0,2)
x4=rnorm(n,0,1.9)
y=rnorm(n,0,2.5)
x=list(x1,x2,x3,x4)
LSE_Reg_per(x,y,s)
```

phi_n *Calculating the value of ϕ function*

Description

phi_n() function gives the value of $\widehat{\phi}(x) = \frac{1}{b_n^2} \frac{\sum_{s=1}^{S} \sum_{r=0}^{m-1} (x - Z_{s+Sr}) \exp\left(-\frac{(x-Z_{s+Sr})^2}{2b_n^2}\right)}{\sum_{s=1}^{S} \sum_{r=0}^{m-1} \exp\left(-\frac{(x-Z_{s+Sr})^2}{2b_n^2}\right)}$ with $b_n = 0.002$.

Usage

```
phi_n(x)
```

Arguments

x	A numeric value.

Value

returns the value of $\widehat{\phi}(x)$

pseudo_gaussian_test *Detecting periodicity of parameters in the regression model*

Description

pseudo_gaussian_test() function gives the value of the statistic test, $T^{(n)}$, for detecting periodicity of parameters in the regression model. See check_periodicity function.

Usage

```
pseudo_gaussian_test(x,z,s)
```

Arguments

x	A list of independent variables with dimension p.
z	The residuals vector.
s	A period of the regression model.

Value

returns the value of the statistic test, $T^{(n)}$.

sd_estimation_for_each_s

Estimating periodic variances in a periodic coefficients regression model

Description

sd_estimation_for_each_s() function gives the estimation of variances, $\widehat{\sigma}_s^2 = \frac{1}{m-p-1}\sum_{r=0}^{m-1}\widehat{\varepsilon}_{s+Sr}^2$ for all $s = 1, ..., S$,in a periodic coefficients regression model.

Usage

```
sd_estimation_for_each_s(x,y,s,beta_hat)
```

Arguments

x	A list of independent variables with dimension p.
y	A response variable.
s	A period of the regression model.
beta_hat	The least squares estimation using LSE_Reg_per.

Value

returns the value of $\widehat{\sigma}_s^2$.

Examples

```
set.seed(6)
n=400
s=4
x1=rnorm(n,0,1.5)
x2=rnorm(n,0,0.9)
x3=rnorm(n,0,2)
x4=rnorm(n,0,1.9)
y=rnorm(n,0,2.5)
x=list(x1,x2,x3,x4)
beta_hat=LSE_Reg_per(x,y,s)$beta
sd_estimation_for_each_s(x,y,s,beta_hat)
```

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