# Package 'NetworkReg'

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Type Package

Title	Generalized Linear Regression Models on Network-Linked Data with
	Statistical Inference

Version 2.0

Date 2024-10-10

**Description** Linear regression model and generalized linear models with nonparametric network effects on network-linked observations. The model is originally proposed by Le and Li (2022) <doi:10.48550/arXiv.2007.00803> and is assumed on observations that are connected by a network or similar relational data structure. A more recent work by Wang, Le and Li (2024) <doi:10.48550/arXiv.2410.01163> further extends the framework to generalized linear models. All these models are implemented in the current package. The model does not assume that the relational data or network structure to be precisely observed; thus, the method is provably robust to a certain level of perturbation of the network structure. The package contains the estimation and inference function for the model.

License GPL (>= 2)

Imports stats, randnet, RSpectra

# NeedsCompilation no

Encoding UTF-8

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NetworkReg-package

Generalized Linear Regression Models on Network-Linked Data with Statistical Inference

#### Description

Linear regression model with nonparametric network effects on network-linked observations. The model is proposed by Le and Li (2022) <arXiv:2007.00803> on observations that are connected by a network or similar relational data structure. The model does not assume that the relational data or network structure to be precisely observed; thus, the method is provably robust to a certain level of perturbation of the network structure. The package contains the estimation and inference function for the model.

#### Details

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#### Author(s)

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#### References

Le, C. M., & Li, T. (2022). Linear regression and its inference on noisy network-linked data. Journal of the Royal Statistical Society Series B: Statistical Methodology, 84(5), 1851-1885.

Wang J, Le C M, Li T. Perturbation-Robust Predictive Modeling of Social Effects by Network Subspace Generalized Linear Models. arXiv preprint arXiv:2410.01163, 2024.

net.gen.from.P generates a network from the given connection probability

#### Description

Generates an adjacency matrix from a given probability matrix, according independent Bernoulli – the so-called inhomogeneous Erdos-Renyi model. It is used to generate new networks from a given model.

# SP.Inf

# Usage

net.gen.from.P(P, mode = "undirected")

#### Arguments

Ρ	connection probability between nodes
mode	"undirected" (default) if the network is undirected, so the adjacency matrix will be symmetric with only upper diagonal entries being generated as independent Bernoulli. Otherwise, the adjacency matrix gives independent Bernoulli every- where.

# Value

An adjacency matrix

SP.Inf

Fitting Generalized Linear Model on Network-Linked Data

# Description

SP.Inf is used to the regression model on network-linked data by subspace project and produce the inference result.

# Usage

SP.Inf(X, Y, A, K, model = "linear", r = NULL, sigma2 = NULL, thr = NULL, alpha.CI = 0.05, boot.thr = TRUE, boot.n = 50)

# Arguments

Х	the covariate matrix where each row is an observation and each column is a covariate. If an intercept is to be included in the model, the column of ones should be in the matrix.
Υ	the column vector of response.
A	the network information. The most natural choice is the adjacency matrix of the network. However, if the network is assumed to be noisy and a better estimate of the structural connection strength, it can also be used. This corresponds to the Phat matrix in the original paper. A Laplacian matrix can also be used, but it should be flipped. See 'Details'.
К	the dimension of the network eigenspace for network effect.
model	the type of Generalized Linear Regression. The "linear", "logistic" and "poisson" represents linear regression, logistic regression and poisson regression. The default is linear regression.

r	the covariate-network cofounding space dimension. This is typically unknown and can be unspecified by using the default value 'NULL'. If so, the user should provide a threshold or resort to a tuning procedure by either the theoretical rule or a bootstrapping method, as described in the paper.
sigma2	the variance of random noise for linear regression. Typically unknown.
thr	threshold for r estimation. If r is unspecified, we will use the thereshold to select r. If this is also 'NULL', aa theoretical threshold or a bootsrapping method can be evoked to estimate it.
alpha.CI	the 1-alpha.CI confidence level will be produced for the parameters.
boot.thr	logical. Only effective if both r and thr are NULLs. If FALSE, the theoretical threshold will be used to select r. Otherwise, the bootstrapping procedure will be used to find the threshold.
boot.n	the number of bootstrapping samples used when boot.thr is TRUE.

# Details

The model fitting procedure is following the paper exactly, so please check the procedure and theory in the paper. If the Laplacian matrix L=D-A is the network quantity to use, notice that typically we treat the smallest values and their corresponding eigenvectors as network cohesive space. Therefore, one should consider flip the Laplacian matrix by using cI - L as the value for A, where c is sufficiently large to ensure PSD of cI-L.

#### Value

A list object with

beta	estimate of beta, the covariate effects
alpha	individual effects
theta	coefficients of confounding effects with respect to the covariates
r	confounding dimension
sigma	estimated random noise variance for linear regression
cov.hat	covariance matrix of beta
coef.mat	beta and the confidence intervals according to alpha.CI and the p-values of the significance test
fitted	fitted value of response
chisq.val	the value of the chi-square statistic for the significance test for network effect
chisq.p	the p-value of the significance test for network effect

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#### SP.Inf

#### References

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#### Examples

```
set.seed(1)
library(randnet)
library(RSpectra)
### Example data generation procedure from Section 5.3 of the paper with logistic regression
n <- 1000
big.model <- BlockModel.Gen(lambda=n^(2/3), n=n, beta=0.2, K=4)</pre>
P <- big.model$P</pre>
big.X <- cbind(rnorm(n), runif(n), rexp(n))</pre>
eigen.P <- eigs_sym(A=P, k=4)</pre>
X.true <- big.X
X.true <- scale(X.true, center=TRUE, scale=TRUE) * sqrt(n/(n-1))</pre>
beta <- matrix(c(1,1,1), ncol=1)</pre>
Xbeta <- X.true %*% beta
U <- eigen.P$vectors[,1:4]</pre>
alpha.coef <- matrix(sqrt(n) * c(1, 1, 1, 1), ncol=1)</pre>
alpha <- U %*% alpha.coef
EY <- (1 + exp(-Xbeta - alpha))^(-1)</pre>
## Model fitting
A <- net.gen.from.P(P)
Y \leq rbinom(n, 1, EY)
fit <- SP.Inf(X.true, Y, A, K=4, model=c("logistic"), alpha=0.05, boot.thr=FALSE)</pre>
fit$coef.mat
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