The goal of the estima function is to estimate the coefficients of the two centered autologistic regression :

$$
\begin{aligned}
logit(p_{i,t}) &= X_{i,t}^T\beta + \beta_{past}\sum_{j\in N_i^{past}} Z_{j,t-1} + \rho_1 \sum_{j\in N_i} Z_{j,t}^{**} + \rho_2 Z_{i,t-1} \\
\Leftrightarrow \quad p_{i,t} &= \frac{exp(X_{i,t}^T\beta + \beta_{past}\sum_{j\in N_i^{past}} Z_{j,t-1} + \rho_1 \sum_{j\in N_i} Z_{j,t}^{**} + \rho_2 Z_{i,t-1})}{1+exp(X_{i,t}^T\beta + \beta_{past}\sum_{j\in N_i^{past}} Z_{j,t-1} + \rho_1 \sum_{j\in N_i} Z_{j,t}^{**} + \rho_2 Z_{i,t-1})}
\end{aligned}
$$

where $Z_{i,t}$ is a binary variable of parameter $p_{i,t}$, N_i is the neighborhood of the site i for the instantaneous spatial dependence, N_i^{past} is the neighborhood of the site i for the spatio-temporal dependence (spread of the illness) and $Z_{i,t-1}^{**}$ is given by :

$$
Z_{i,t}^{**} = Z_{i,t} - \frac{exp(X_{i,t}^T\beta + \beta_{past}\sum_{j\in N_i^{past}} Z_{j,t-1} + \rho_2 Z_{i,t-1})}{1+exp(X_{i,t}^T\beta + \beta_{past}\sum_{j\in N_i^{past}} Z_{j,t-1} + \rho_2 Z_{i,t-1})}.
$$

Estimation uses the pseudo-likelihood :

$$
\mathcal{L}(\beta, \beta_{past}, \rho_1, \rho_2) = \prod_{t=1}^{T} \prod_{1\leq i\leq n} (p_{i,t})^{z_{i,t}} (1-p_{i,t})^{1-z_{i,t}}.
$$

For more detail see Gegout-Petit, Guérin-Dubrana, Li, 2019.

The parameters of spatio-temporal dependence ρ_1, ρ_2, β_{past} can be interpreted as practical biological processes :

- Instantaneous spatial dependence ρ_1. It quantifies the spatial autocorrelation between neighbours for the occurence of the event at each time t,
- Temporal dependence ρ_2. It quantifies the temporal dependence on the previous year's status,
- Coefficient β_{past} : it quantifies the spread of the illness coming from the previous year's status of the neighbours

The function "estima" estimates the parameters with different possibilities for β_{past} and $\sum_{j\in N_i^{past}} Z_{j,t-1}$:

if "covpast = FALSE : estimates the parameter $\beta = \begin{pmatrix}\beta_0\\ \beta_1\\ \beta_2\\ \beta_3\end{pmatrix}$ and $X_{i,t}^T = \begin{pmatrix}1\\ x_{i,t}^1\\ x_{i,t}^2\\ x_{i,t}^3\end{pmatrix}$ where $x_{i,t}^j \forall j \in (1,2,3)$ is a spatio-temporal covariate. There can be 0, 1, 2 or 3 covariates. In this case, there is no regression on $\sum_{j\in N_i^{past}} Z_{j,t-1}$ ($\beta_{past} = 0$).

if "covpast = TRUE" : the function estimates the parameters $\beta = \begin{pmatrix}\beta_0\\ \beta_1\\ \beta_2\\ \beta_3\end{pmatrix}$ and β_{past}.