Package 'Sim.DiffProc'

January 20, 2025

Type Package

Version 4.9

Date 2024-03-06

Title Simulation of Diffusion Processes

Author Arsalane Chouaib Guidoum [cre, aut]
(<https://orcid.org/0000-0003-3781-2160>),
Kamal Boukhetala [aut]

Maintainer Arsalane Chouaib Guidoum <acguidoum@univ-tam.dz>

Depends R (>= 4.0.0)

Imports Deriv (>= 3.8.0), MASS (>= 7.3-30), parallel

Suggests deSolve (>= 1.11), knitr (>= 1.10), rgl (>= 0.93.991),
rmarkdown (>= 0.8), scatterplot3d (>= 0.3-36), sm (>= 2.2-5.3)

BugReports https://github.com/acguidoum/Sim.DiffProc/issues

URL https://github.com/acguidoum/Sim.DiffProc

VignetteBuilder knitr

Encoding UTF-8

Description It provides users with a wide range of tools to simulate, estimate, analyze, and visualize the dynamics of stochastic differential systems in both forms Ito and Stratonovich. Statistical analysis with parallel Monte Carlo and moment equations methods of SDEs <doi:10.18637/jss.v096.i02>. Enabled many searchers in different domains to use these equations to modeling practical problems in financial and actuarial modeling and other areas of application, e.g., modeling and simulate of first passage time problem in shallow water using the attractive center (Boukhetala K, 1996) ISBN:1-56252-342-2.

Classification/MSC 37H10, 37M10, 60H05, 60H10, 60H35, 60J60, 65C05,
68N15, 68Q10

NeedsCompilation yes

Repository CRAN

Date/Publication 2024-03-06 01:00:19 UTC

Contents

`Sim.DiffProc-package` *Simulation of Diffusion Processes*

Description

It provides users with a wide range of tools to simulate, estimate, analyze, and visualize the dynamics of stochastic differential systems in both forms Ito and Stratonovich. Statistical analysis with parallel Monte Carlo and moment equations methods of SDEs <doi:10.18637/jss.v096.i02>. Enabled many searchers in different domains to use these equations to modeling practical problems in financial and actuarial modeling and other areas of application, e.g., modeling and simulate of first passage time problem in shallow water using the attractive center (Boukhetala K, 1996) ISBN:1-56252-342-2.

Details

Package:	Sim.DiffProc
Type:	Package
Version:	4.9
Date:	2024-03-06

License:	GPL (>= 2)
Depends:	R (>= 4.0.0)
Imports:	Deriv (>= 3.8.0), MASS (>= 7.3-30), parallel
Suggests:	deSolve (>= 1.11), knitr (>= 1.10), rgl (>= 0.93.991), rmarkdown (>= 0.8), scatterplot3d (>= 0.3-36), s
Classification/MSC:	37H10, 37M10, 60H05, 60H10, 60H35, 60J60, 65C05, 68N15, 68Q10

There are main types of functions in this package:

1. Simulation of solution to 1,2 and 3-dim stochastic differential equations of Itô and Stratonovich types, with different methods.
2. Simulation of solution to 1,2 and 3-dim diffusion bridge of Itô and Stratonovich types, with different methods.
3. Simulation the first-passage-time (f.p.t) in 1,2 and 3-dim sde of Itô and Stratonovich types.
4. Calculate symbolic ODE's of moment equations (means and variances-covariance) for 1,2 and 3-dim SDE's.
5. Monte-Carlo replicates of a statistic applied to 1,2 and 3-dim SDE's at any time t.
6. Computing the basic statistics (mean, var, median, ...) of the processes at any time t using the Monte Carlo method.
7. Random number generators (RN's) to generate 1,2 and 3-dim sde of Itô and Stratonovich types.
8. Approximate the transition density 1,2 and 3-dim of the processes at any time t.
9. Approximate the density of first-passage-time in 1,2 and 3-dim SDE's.
10. Computing the Itô and Stratonovich stochastic integrals.
11. Estimate drift and diffusion parameters by the method of maximum pseudo-likelihood of the one-dim stochastic differential equation.
12. Converting Sim.DiffProc objects to LaTeX.
13. Displaying an object inheriting from class `"sde"` (1,2 and 3 dim).

For more examples see `demo(Sim.DiffProc)`, and for an overview of this package, see `browseVignettes('Sim.DiffProc'` for more informations.

Requirements

R version >= 4.0.0

Citation

Guidoum AC, Boukhetala K (2020). "Performing Parallel Monte Carlo and Moment Equations Methods for Itô and Stratonovich Stochastic Differential Systems: R Package Sim.DiffProc". *Journal of Statistical Software*, **96**(2), 1–82. doi:10.18637/jss.v096.i02

Licence

This package and its documentation are usable under the terms of the "GNU General Public License", a copy of which is distributed with the package.

Author(s)

A.C. Guidoum <acguidoum@univ-tam.dz> and K. Boukhetala <kboukhetala@usthb.dz>.
Please send comments, error reports, etc. to the author via the addresses email.

References

Argyrakisa, P. and G.H. Weiss (2006). A first-passage time problem for many random walkers. *Physica A*. **363**, 343–347.

Aytug H., G. J. Koehler (2000). New stopping criterion for genetic algorithms. *European Journal of Operational Research*, **126**, 662–674.

Boukhetala, K. (1994). Simulation study of a dispersion about an attractive centre. In proceedings of 11th Symposium Computational Statistics, edited by R.Dutter and W.Grossman, Wien , Austria, 128–130.

Boukhetala, K. (1996). Modelling and simulation of a dispersion pollutant with attractive centre. ed by Computational Mechanics Publications, Southampton ,U.K and Computational Mechanics Inc, Boston, USA, 245–252.

Boukhetala, K. (1998a). Estimation of the first passage time distribution for a simulated diffusion process. *Maghreb Math.Rev*, **7**(1), 1–25.

Boukhetala, K. (1998b). Kernel density of the exit time in a simulated diffusion. *les Annales Maghrebines De L ingenieur*, **12**, 587–589.

Ding, M. and G. Rangarajan. (2004). First Passage Time Problem: A Fokker-Planck Approach. *New Directions in Statistical Physics*. ed by L. T. Wille. Springer. 31–46.

Ait-Sahalia, Y. (1999). Transition densities for interest rate and other nonlinear diffusions. *The Journal of Finance*, 54, 1361–1395.

Ait-Sahalia, Y. (2002). Maximum likelihood estimation of discretely sampled diffusions: a closed-form approximation approach. *Econometrica*. 70, 223–262.

Roman, R.P., Serrano, J. J., Torres, F. (2008). First-passage-time location function: Application to determine first-passage-time densities in diffusion processes. *Computational Statistics and Data Analysis*. **52**, 4132–4146.

Roman, R.P., Serrano, J. J., Torres, F. (2012). An R package for an efficient approximation of first-passage-time densities for diffusion processes based on the FPTL function. *Applied Mathematics and Computation*, **218**, 8408–8428.

Kessler, M. (1997). Estimation of an ergodic diffusion from discrete observations. *Scand. J. Statist.*, 24, 211-229.

Gardiner, C. W. (1997). *Handbook of Stochastic Methods*. Springer-Verlag, New York.

Friedman, A. (1975). *Stochastic differential equations and applications*. Volume 1, ACADEMIC PRESS.

Henderson, D. and Plaschko,P. (2006). *Stochastic differential equations in science and engineering*. World Scientific.

Croissant, Y. (2014). Ecdat: Data sets for econometrics. R *package version 0.2-5*.

Vasicek, O. (1977). An Equilibrium Characterization of the Term Structure. *Journal of Financial Economics*, 5, 177–188.

Allen, E. (2007). *Modeling with Itô stochastic differential equations*. Springer-Verlag.

Jedrzejewski, F. (2009). *Modeles aleatoires et physique probabiliste*. Springer-Verlag.

Iacus, S.M. (2008). *Simulation and inference for stochastic differential equations: with R examples*. Springer-Verlag, New York.

Iacus, S.M. (2014). sde: Simulation and Inference for Stochastic Differential Equations. R *package version 2.0.13*.

Brouste, A. et al. (2014). The yuima Project: A Computational Framework for Simulation and Inference of Stochastic Differential Equations. *Journal of Statistical Software*, **57**(4).

Kloeden, P.E, and Platen, E. (1989). A survey of numerical methods for stochastic differential equations. *Stochastic Hydrology and Hydraulics*, **3**, 155–178.

Kloeden, P.E, and Platen, E. (1991a). Relations between multiple Itô and stratonovich integrals. *Stochastic Analysis and Applications*, **9**(3), 311–321.

Kloeden, P.E, and Platen, E. (1991b). Stratonovich and Itô stochastic taylor expansions. *Mathematische Nachrichten*, **151**, 33–50.

Kloeden, P.E, and Platen, E. (1995). *Numerical Solution of Stochastic Differential Equations*. Springer-Verlag, New York.

Oksendal, B. (2000). *Stochastic Differential Equations: An Introduction with Applications*. 5th edn. Springer-Verlag, Berlin.

B.L.S. Prakasa Rao. (1999). *Statistical Inference for Diffusion Type Processes*. Arnold, London and Oxford University press, New York.

Kutoyants, Y.A. (2004). *Statistical Inference for Ergodic Diffusion Processes*. Springer, London.

Sorensen, H. (2000). Inference for Diffusion Processes and Stochastic Volatility Models. Ph.D. thesis, Department of Mathematical Sciences, University of Copenhagen.

Sorensen, H. (2002). Estimation of diffusion parameters for discretely observed diffusion processes. *Bernoulli*, 8, 491–508.

Sorensen, H. (2004). Parametric inference for diffusion processes observed at discrete points in time: a survey. *International Statistical Review*, 72, 337–354.

Platen, E. (1980). Weak convergence of approximations of Itô integral equations. *Z Angew Math Mech*. **60**, 609–614.

Platen, E. and Bruti-Liberati, N. (2010). *Numerical Solution of Stochastic Differential Equations with Jumps in Finance.* Springer-Verlag, New York.

Itô, Y, and Mitsui, T. (1993). Simulation of Stochastic Differential Equations. *The Annals of the Institute of Statistical Mathematics*, **3**, 419–432.

Risken, H. (2001). *The Fokker Planck Equation : Methods of Solutions and Applications*. 2nd edition, Springer Series in Synergetics.

Dacunha, D.C. and Florens, D.Z. (1986). Estimation of the Coefficients of a Diffusion from Discrete Observations. *Stochastics*. 19, 263–284.

Dohnal, G. (1987). On estimating the diffusion coefficient. *J. Appl.Prob.*, 24, 105–114.

Genon, V.C. (1990). Maximum constrast estimation for diffusion processes from discrete observation. *Statistics*, 21, 99–116.

Protter, P. (2005). *Stochastic Integration and Differential Equations.* 2nd edn. Springer-Verlag, New York.

Bladt, M. and Sorensen, M. (2007). Simple simulation of diffusion bridges with application to likelihood inference for diffusions. *Working Paper, University of Copenhagen*.

Ozaki, T. (1992). A bridge between nonlinear time series models and nonlinear stochastic dynamical systems: A local linearization approach. *Statistica Sinica*, 2, 25-83.

Shoji, L., Ozaki, T. (1998). Estimation for nonlinear stochastic differential equations by a local linearization method. *Stochastic Analysis and Applications*, 16, 733-752.

Nicolau, J. (2004). Introduction to the estimation of stochastic differential equations based on discrete observations. *Autumn School and International Conference, Stochastic Finance*.

F C Klebaner, F.C. (2005). *Introduction to stochastic calculus with application.* 2nd edn. Imperial College Press (ICP).

Henderson, D. and Plaschko, P. (2006). *Stochastic differential equations in science and engineering.* World Scientific.

See Also

sde, yumia, DiffusionRgqd, DiffusionRjgqd, DiffusionRimp, QPot, diffeqr, fptdApprox.

`BM` *Brownian motion, Brownian bridge, geometric Brownian motion, and arithmetic Brownian motion simulators*

Description

The (S3) generic function for simulation of brownian motion, brownian bridge, geometric brownian motion, and arithmetic brownian motion.

Usage

```
BM(N, ...)
BB(N, ...)
GBM(N, ...)
ABM(N, ...)

## Default S3 method:
BM(N =1000,M=1,x0=0,t0=0,T=1,Dt=NULL, ...)
## Default S3 method:
BB(N =1000,M=1,x0=0,y=0,t0=0,T=1,Dt=NULL, ...)
## Default S3 method:
GBM(N =1000,M=1,x0=1,t0=0,T=1,Dt=NULL,theta=1,sigma=1, ...)
## Default S3 method:
ABM(N =1000,M=1,x0=0,t0=0,T=1,Dt=NULL,theta=1,sigma=1, ...)
```

Arguments

N	number of simulation steps.
M	number of trajectories.
x0	initial value of the process at time t_0.
y	terminal value of the process at time T of the BB.
t0	initial time.
T	final time.
Dt	time step of the simulation (discretization). If it is NULL a default $\Delta t = \frac{T-t_0}{N}$.
theta	the interest rate of the ABM and GBM.
sigma	the volatility of the ABM and GBM.
...	potentially further arguments for (non-default) methods.

Details

The function BM returns a trajectory of the **standard Brownian motion** (Wiener process) in the time interval $[t_0, T]$. Indeed, for $W(dt)$ it holds true that $W(dt) \rightarrow W(dt) - W(0) \rightarrow \mathcal{N}(0, dt)$, where $\mathcal{N}(0, 1)$ is normal distribution Normal.

The function BB returns a trajectory of the **Brownian bridge** starting at x_0 at time t_0 and ending at y at time T; i.e., the diffusion process solution of stochastic differential equation:

$$dX_t = \frac{y - X_t}{T - t} dt + dW_t$$

The function GBM returns a trajectory of the **geometric Brownian motion** starting at x_0 at time t_0; i.e., the diffusion process solution of stochastic differential equation:

$$dX_t = \theta X_t dt + \sigma X_t dW_t$$

The function ABM returns a trajectory of the **arithmetic Brownian motion** starting at x_0 at time t_0; i.e.,; the diffusion process solution of stochastic differential equation:

$$dX_t = \theta dt + \sigma dW_t$$

Value

X	an visible ts object.

Author(s)

A.C. Guidoum, K. Boukhetala.

References

Allen, E. (2007). *Modeling with Ito stochastic differential equations.* Springer-Verlag, New York.

Jedrzejewski, F. (2009). *Modeles aleatoires et physique probabiliste.* Springer-Verlag, New York.

Henderson, D and Plaschko, P. (2006). *Stochastic differential equations in science and engineering.* World Scientific.

See Also

This functions BM, BBridge and GBM are available in other packages such as "sde".

Examples

```
op <- par(mfrow = c(2, 2))

## Brownian motion
set.seed(1234)
X <- BM(M = 100)
plot(X,plot.type="single")
lines(as.vector(time(X)),rowMeans(X),col="red")

## Brownian bridge
set.seed(1234)
X <- BB(M =100)
plot(X,plot.type="single")
lines(as.vector(time(X)),rowMeans(X),col="red")

## Geometric Brownian motion
set.seed(1234)
X <- GBM(M = 100)
plot(X,plot.type="single")
lines(as.vector(time(X)),rowMeans(X),col="red")

## Arithmetic Brownian motion
set.seed(1234)
X <- ABM(M = 100)
plot(X,plot.type="single")
lines(as.vector(time(X)),rowMeans(X),col="red")

par(op)
```

---

bridgesde1d *Simulation of 1-D Bridge SDE*

Description

The (S3) generic function bridgesde1d for simulation of 1-dim bridge stochastic differential equations,Itô or Stratonovich type, with different methods.

Usage

```
bridgesde1d(N, ...)
## Default S3 method:
bridgesde1d(N = 1000, M=1, x0 = 0, y = 0, t0 = 0, T = 1, Dt,
   drift, diffusion, alpha = 0.5, mu = 0.5, type = c("ito", "str"),
   method = c("euler", "milstein", "predcorr", "smilstein", "taylor",
   "heun", "rk1", "rk2", "rk3"), ...)
```

```
## S3 method for class 'bridgesde1d'
summary(object, at ,digits=NULL, ...)
## S3 method for class 'bridgesde1d'
time(x, ...)
## S3 method for class 'bridgesde1d'
mean(x, at, ...)
## S3 method for class 'bridgesde1d'
Median(x, at, ...)
## S3 method for class 'bridgesde1d'
Mode(x, at, ...)
## S3 method for class 'bridgesde1d'
quantile(x, at, ...)
## S3 method for class 'bridgesde1d'
kurtosis(x, at, ...)
## S3 method for class 'bridgesde1d'
skewness(x, at, ...)
## S3 method for class 'bridgesde1d'
min(x, at, ...)
## S3 method for class 'bridgesde1d'
max(x, at, ...)
## S3 method for class 'bridgesde1d'
moment(x, at, ...)
## S3 method for class 'bridgesde1d'
cv(x, at, ...)
## S3 method for class 'bridgesde1d'
bconfint(x, at,  ...)

## S3 method for class 'bridgesde1d'
plot(x, ...)
## S3 method for class 'bridgesde1d'
lines(x, ...)
## S3 method for class 'bridgesde1d'
points(x, ...)
```

Arguments

N	number of simulation steps.
M	number of trajectories.
x0	initial value of the process at time t0.
y	terminal value of the process at time T.
t0	initial time.
T	final time.
Dt	time step of the simulation (discretization). If it is missing a default $\Delta t = \frac{T-t_0}{N}$.
drift	drift coefficient: an expression of two variables t and x.

diffusion	diffusion coefficient: an expression of two variables t and x.
alpha, mu	weight of the predictor-corrector scheme; the default alpha = 0.5 and mu = 0.5.
type	if type="ito" simulation diffusion bridge of Itô type, else type="str" simulation diffusion bridge of Stratonovich type; the default type="ito".
method	numerical methods of simulation, the default method = "euler"; see snssde1d.
x, object	an object inheriting from class "bridgesde1d".
at	time between t0 and T. Monte-Carlo statistics of the solution X_t at time at. The default at = T/2.
digits	integer, used for number formatting.
...	potentially further arguments for (non-default) methods.

Details

The function bridgesde1d returns a trajectory of the diffusion bridge starting at x at time t0 and ending at y at time T.

The methods of approximation are classified according to their different properties. Mainly two criteria of optimality are used in the literature: the strong and the weak (orders of) convergence. The method of simulation can be one among: Euler-Maruyama Order 0.5, Milstein Order 1, Milstein Second-Order, Predictor-Corrector method, Itô-Taylor Order 1.5, Heun Order 2 and Runge-Kutta Order 1, 2 and 3.

An overview of this package, see browseVignettes('Sim.DiffProc') for more informations.

Value

bridgesde1d returns an object inheriting from class "bridgesde1d".

X	an invisible ts object.
drift	drift coefficient.
diffusion	diffusion coefficient.
C	indices of crossing realized of X(t).
type	type of sde.
method	the numerical method used.

Author(s)

A.C. Guidoum, K. Boukhetala.

References

Bladt, M. and Sorensen, M. (2007). Simple simulation of diffusion bridges with application to likelihood inference for diffusions. *Working Paper, University of Copenhagen*.

Iacus, S.M. (2008). *Simulation and inference for stochastic differential equations: with R examples*. Springer-Verlag, New York

See Also

bridgesde2d and bridgesde3d for 2 and 3-dim.

DBridge in package "sde".

Examples

```
## Example 1: Ito bridge sde
## Ito Bridge sde
## dX(t) = 2*(1-X(t)) *dt + dW(t)
## x0 = 2 at time t0=0 , and y = 1 at time T=1
set.seed(1234)

f <- expression( 2*(1-x) )
g <- expression( 1 )
mod1 <- bridgesde1d(drift=f,diffusion=g,x0=2,y=1,M=1000)
mod1
summary(mod1) ## Monte-Carlo statistics at T/2=0.5
summary(mod1,at=0.75) ## Monte-Carlo statistics at 0.75
## Not run:
plot(mod1)
lines(time(mod1),apply(mod1$X,1,mean),col=2,lwd=2)
lines(time(mod1),apply(mod1$X,1,bconfint,level=0.95)[1,],col=4,lwd=2)
lines(time(mod1),apply(mod1$X,1,bconfint,level=0.95)[2,],col=4,lwd=2)
legend("topleft",c("mean path",paste("bound of", 95," percent confidence")),
       inset = .01,col=c(2,4),lwd=2,cex=0.8)

## End(Not run)

## Example 2: Stratonovich sde
## dX(t) = ((2-X(t))/(2-t)) dt + X(t) o dW(t)
## x0 = 2 at time t0=0 , and y = 2 at time T=1
set.seed(1234)

f <- expression((2-x)/(2-t))
g <- expression(x)
mod2 <- bridgesde1d(type="str",drift=f,diffusion=g,M=1000,x0=2,y=2)
mod2
summary(mod2,at = 0.25) ## Monte-Carlo statistics at 0.25
summary(mod2,at = 0.5)  ## Monte-Carlo statistics at 0.5
summary(mod2,at = 0.75 )## Monte-Carlo statistics at 0.75
## Not run:
plot(mod2)
lines(time(mod2),apply(mod2$X,1,mean),col=2,lwd=2)
lines(time(mod2),apply(mod2$X,1,bconfint,level=0.95)[1,],col=4,lwd=2)
lines(time(mod2),apply(mod2$X,1,bconfint,level=0.95)[2,],col=4,lwd=2)
legend("topright",c("mean path",paste("bound of", 95," percent confidence")),
       inset = .01,col=c(2,4),lwd=2,cex=0.8)

## End(Not run)
```

bridgesde2d *Simulation of 2-D Bridge SDE's*

Description

The (S3) generic function `bridgesde2d` for simulation of 2-dim bridge stochastic differential equations,Itô or Stratonovich type, with different methods.

Usage

```
bridgesde2d(N, ...)
## Default S3 method:
bridgesde2d(N = 1000, M = 1, x0 = c(0, 0),
   y = c(0, 0),t0 = 0, T = 1, Dt,drift, diffusion, corr = NULL,
   alpha = 0.5, mu = 0.5,type = c("ito", "str"),method =
   c("euler", "milstein","predcorr", "smilstein", "taylor",
   "heun", "rk1", "rk2", "rk3"), ...)

## S3 method for class 'bridgesde2d'
summary(object, at,
     digits=NULL, ...)
## S3 method for class 'bridgesde2d'
time(x, ...)
## S3 method for class 'bridgesde2d'
mean(x, at, ...)
## S3 method for class 'bridgesde2d'
Median(x, at, ...)
## S3 method for class 'bridgesde2d'
Mode(x, at, ...)
## S3 method for class 'bridgesde2d'
quantile(x, at, ...)
## S3 method for class 'bridgesde2d'
kurtosis(x, at, ...)
## S3 method for class 'bridgesde2d'
skewness(x, at, ...)
## S3 method for class 'bridgesde2d'
min(x, at, ...)
## S3 method for class 'bridgesde2d'
max(x, at, ...)
## S3 method for class 'bridgesde2d'
moment(x, at, ...)
## S3 method for class 'bridgesde2d'
cv(x, at, ...)
## S3 method for class 'bridgesde2d'
bconfint(x, at, ...)

## S3 method for class 'bridgesde2d'
```

```
plot(x, ...)
## S3 method for class 'bridgesde2d'
lines(x, ...)
## S3 method for class 'bridgesde2d'
points(x, ...)
## S3 method for class 'bridgesde2d'
plot2d(x, ...)
## S3 method for class 'bridgesde2d'
lines2d(x, ...)
## S3 method for class 'bridgesde2d'
points2d(x, ...)
```

Arguments

`N`	number of simulation steps.
`M`	number of trajectories.
`x0`	initial value (numeric vector of length 2) of the process X_t and Y_t at time t_0.
`y`	terminal value (numeric vector of length 2) of the process X_t and Y_t at time T.
`t0`	initial time.
`T`	final time.
`Dt`	time step of the simulation (discretization). If it is `missing` a default $\Delta t = \frac{T-t_0}{N}$.
`drift`	drift coefficient: an `expression` of three variables `t`, `x` and `y` for process X_t and Y_t.
`diffusion`	diffusion coefficient: an `expression` of three variables `t`, `x` and `y` for process X_t and Y_t.
`corr`	the correlation structure of two Brownian motions W1(t) and W2(t); must be a real symmetric positive-definite square matrix of dimension 2.
`alpha, mu`	weight of the predictor-corrector scheme; the default `alpha = 0.5` and `mu = 0.5`.
`type`	if `type="ito"` simulation diffusion bridge of Itô type, else `type="str"` simulation diffusion bridge of Stratonovich type; the default `type="ito"`.
`method`	numerical methods of simulation, the default `method = "euler"`; see `snssde2d`.
`x, object`	an object inheriting from class `"bridgesde2d"`.
`at`	time between `t0` and `T`. Monte-Carlo statistics of the solution (X_t, Y_t) at time `at`. The default `at = T/2`.
`digits`	integer, used for number formatting.
`...`	potentially further arguments for (non-default) methods.

Details

The function `bridgesde2d` returns a `mts` of the diffusion bridge starting at `x` at time `t0` and ending at `y` at time `T`. W1(t) and W2(t) are two standard Brownian motion independent if `corr=NULL`.

The methods of approximation are classified according to their different properties. Mainly two criteria of optimality are used in the literature: the strong and the weak (orders of) convergence.

The method of simulation can be one among: `Euler-Maruyama Order 0.5`, `Milstein Order 1`, `Milstein Second-Order`, `Predictor-Corrector method`, `Itô-Taylor Order 1.5`, `Heun Order 2` and `Runge-Kutta Order 1, 2 and 3`.

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

Value

`bridgesde2d` returns an object inheriting from `class` `"bridgesde2d"`.

X, Y	an invisible `mts` (2-dim) object (X(t),Y(t)).
driftx, drifty	drift coefficient of X(t) and Y(t).
diffx, diffy	diffusion coefficient of X(t) and Y(t).
Cx, Cy	indices of crossing realized of X(t) and Y(t).
type	type of sde.
method	the numerical method used.

Author(s)

A.C. Guidoum, K. Boukhetala.

References

Bladt, M. and Sorensen, M. (2007). Simple simulation of diffusion bridges with application to likelihood inference for diffusions. *Working Paper, University of Copenhagen*.

Iacus, S.M. (2008). *Simulation and inference for stochastic differential equations: with R examples*. Springer-Verlag, New York

See Also

`bridgesde1d` for simulation of 1-dim SDE.

`DBridge` in package "sde".

Examples

```
## dX(t) = 4*(-1-X(t)) dt + 0.2 dW1(t)
## dY(t) = X(t) dt + 0 dW2(t)
## x01 = 0 , y01 = 0
## x02 = 0, y02 = 0
## W1(t) and W2(t) two correlated Brownian motion with matrix Sigma=matrix(c(1,0.7,0.7,1),nrow=2)
set.seed(1234)

fx <- expression(4*(-1-x) , x)
gx <- expression(0.2 , 0)
Sigma= matrix(c(1,0.7,0.7,1),nrow=2)
res <- bridgesde2d(drift=fx,diffusion=gx,Dt=0.005,M=500,corr=Sigma)
res
summary(res) ## Monte-Carlo statistics at time T/2=2.5
summary(res,at=1) ## Monte-Carlo statistics at time 1
summary(res,at=4) ## Monte-Carlo statistics at time 4
```

```
##
plot(res,type="n")
lines(time(res),apply(res$X,1,mean),col=3,lwd=2)
lines(time(res),apply(res$Y,1,mean),col=4,lwd=2)
legend("topright",c(expression(E(X[t])),expression(E(Y[t]))),lty=1,inset = .7,col=c(3,4))
##
plot2d(res)
```

---

bridgesde3d *Simulation of 3-D Bridge SDE's*

Description

The (S3) generic function `bridgesde3d` for simulation of 3-dim bridge stochastic differential equations,Itô or Stratonovich type, with different methods.

Usage

```
bridgesde3d(N, ...)
## Default S3 method:
bridgesde3d(N=1000,M=1, x0=c(0,0,0),
   y=c(0,0,0), t0 = 0, T = 1, Dt, drift, diffusion, corr = NULL,
   alpha = 0.5, mu = 0.5,type = c("ito", "str"), method =
   c("euler", "milstein","predcorr","smilstein", "taylor",
"heun","rk1", "rk2", "rk3"), ...)

## S3 method for class 'bridgesde3d'
summary(object, at,
     digits=NULL, ...)
## S3 method for class 'bridgesde3d'
time(x, ...)
## S3 method for class 'bridgesde3d'
mean(x, at, ...)
## S3 method for class 'bridgesde3d'
Median(x, at, ...)
## S3 method for class 'bridgesde3d'
Mode(x, at, ...)
## S3 method for class 'bridgesde3d'
quantile(x, at, ...)
## S3 method for class 'bridgesde3d'
kurtosis(x, at, ...)
## S3 method for class 'bridgesde3d'
skewness(x, at, ...)
## S3 method for class 'bridgesde3d'
min(x, at, ...)
## S3 method for class 'bridgesde3d'
max(x, at, ...)
```

```
## S3 method for class 'bridgesde3d'
moment(x, at, ...)
## S3 method for class 'bridgesde3d'
cv(x, at, ...)
## S3 method for class 'bridgesde3d'
bconfint(x, at, ...)

## S3 method for class 'bridgesde3d'
plot(x, ...)
## S3 method for class 'bridgesde3d'
lines(x, ...)
## S3 method for class 'bridgesde3d'
points(x, ...)
## S3 method for class 'bridgesde3d'
plot3D(x, display = c("persp","rgl"), ...)
```

Arguments

N	number of simulation steps.
M	number of trajectories.
x0	initial value (numeric vector of length 3) of the process X_t, Y_t and Z_t at time t_0.
y	terminal value (numeric vector of length 3) of the process X_t, Y_t and Z_t at time T.
t0	initial time.
T	final time.
Dt	time step of the simulation (discretization). If it is `missing` a default $\Delta t = \frac{T-t_0}{N}$.
drift	drift coefficient: an `expression` of four variables `t`, `x`, `y` and `z` for process X_t, Y_t and Z_t.
diffusion	diffusion coefficient: an `expression` of four variables `t`, `x`, `y` and `z` for process X_t, Y_t and Z_t.
corr	the correlation structure of three Brownian motions W1(t), W2(t) and W3(t); must be a real symmetric positive-definite square matrix of dimension 3.
alpha	weight `alpha` of the predictor-corrector scheme; the default `alpha = 0.5`.
mu	weight `mu` of the predictor-corrector scheme; the default `mu = 0.5`.
type	if `type="ito"` simulation diffusion bridge of Itô type, else `type="str"` simulation diffusion bridge of Stratonovich type; the default `type="ito"`.
method	numerical methods of simulation, the default `method = "euler"`; see `snssde3d`.
x, object	an object inheriting from class `"bridgesde3d"`.
at	time between `t0` and `T`. Monte-Carlo statistics of the solution (X_t, Y_t, Z_t) at time `at`. The default `at = T/2`.
digits	integer, used for number formatting.
display	`"persp"` perspective and `"rgl"` plots.
...	potentially further arguments for (non-default) methods.

Details

The function bridgesde3d returns a mts of the diffusion bridge starting at x at time t0 and ending at y at time T. W1(t), W2(t) and W3(t) three standard Brownian motion independent if corr=NULL.

The methods of approximation are classified according to their different properties. Mainly two criteria of optimality are used in the literature: the strong and the weak (orders of) convergence. The method of simulation can be one among: Euler-Maruyama Order 0.5, Milstein Order 1, Milstein Second-Order, Predictor-Corrector method, Itô-Taylor Order 1.5, Heun Order 2 and Runge-Kutta Order 1, 2 and 3.

An overview of this package, see browseVignettes('Sim.DiffProc') for more informations.

Value

bridgesde3d returns an object inheriting from class "bridgesde3d".

X, Y, Z	an invisible mts (3-dim) object (X(t),Y(t),Z(t)).
driftx, drifty, driftz	drift coefficient of X(t), Y(t) and Z(t).
diffx, diffy, diffz	diffusion coefficient of X(t), Y(t) and Z(t).
Cx, Cy, Cz	indices of crossing realized of X(t), Y(t)) and Z(t).
type	type of sde.
method	the numerical method used.

Author(s)

A.C. Guidoum, K. Boukhetala.

References

Bladt, M. and Sorensen, M. (2007). Simple simulation of diffusion bridges with application to likelihood inference for diffusions. *Working Paper, University of Copenhagen*.

Iacus, S.M. (2008). *Simulation and inference for stochastic differential equations: with R examples*. Springer-Verlag, New York

See Also

bridgesde1d for simulation of 1-dim SDE. DBridge in package "sde".

bridgesde2d for simulation of 2-dim SDE.

Examples

```
## dX(t) = 4*(-1-X(t))*Y(t) dt + 0.2 * dW1(t) ; x01 = 0 and y01 = 0
## dY(t) = 4*(1-Y(t)) *X(t) dt + 0.2 * dW2(t) ; x02 = -1 and y02 = -2
## dZ(t) = 4*(1-Z(t)) *Y(t) dt + 0.2 * dW3(t) ; x03 = 0.5 and y03 = 0.5
## W1(t), W2(t) and W3(t) are three correlated Brownian motions with Sigma
set.seed(1234)
```

```
fx <- expression(4*(-1-x)*y, 4*(1-y)*x, 4*(1-z)*y)
gx <- rep(expression(0.2),3)
# correlation matrix
Sigma <-matrix(c(1,0.3,0.5,0.3,1,0.2,0.5,0.2,1),nrow=3,ncol=3)

res <- bridgesde3d(x0=c(0,-1,0.5),y=c(0,-2,0.5),drift=fx,diffusion=gx,corr=Sigma,M=200)
res
summary(res) ## Monte-Carlo statistics at time T/2=0.5
summary(res,at=0.25) ## Monte-Carlo statistics at time 0.25
summary(res,at=0.75) ## Monte-Carlo statistics at time 0.75
##
plot(res,type="n")
lines(time(res),apply(res$X,1,mean),col=3,lwd=2)
lines(time(res),apply(res$Y,1,mean),col=4,lwd=2)
lines(time(res),apply(res$Z,1,mean),col=5,lwd=2)
legend("topleft",c(expression(E(X[t])),expression(E(Y[t])),
       expression(E(Z[t]))),lty=1,inset = .01,col=c(3,4,5))
##
plot3D(res,display = "persp",main="3-dim bridge sde")
```

`fitsde` *Maximum Pseudo-Likelihood Estimation of 1-D SDE*

Description

The (S3) generic function "`fitsde`" of estimate drift and diffusion parameters by the method of maximum pseudo-likelihood of the 1-dim stochastic differential equation.

Usage

```
fitsde(data, ...)
## Default S3 method:
fitsde(data, drift, diffusion, start = list(), pmle = c("euler","kessler",
   "ozaki", "shoji"), optim.method = "L-BFGS-B",
   lower = -Inf, upper = Inf, ...)

## S3 method for class 'fitsde'
summary(object, ...)
## S3 method for class 'fitsde'
coef(object, ...)
## S3 method for class 'fitsde'
vcov(object, ...)
## S3 method for class 'fitsde'
logLik(object, ...)
## S3 method for class 'fitsde'
AIC(object, ...)
## S3 method for class 'fitsde'
BIC(object, ...)
## S3 method for class 'fitsde'
confint(object,parm, level=0.95, ...)
```

Arguments

`data`	a univariate time series (`ts` class).
`drift`	drift coefficient: an `expression` of two variables `t`, `x` and `theta` a vector of parameters of sde. See Examples.
`diffusion`	diffusion coefficient: an `expression` of two variables `t`, `x` and `theta` a vector of parameters of sde. See Examples.
`start`	named list of starting values for optimizer. See Examples.
`pmle`	a `character` string specifying the method; can be either: `"euler"` (Euler pseudo-likelihood), `"ozaki"` (Ozaki pseudo-likelihood), `"shoji"` (Shoji pseudo-likelihood), and `"kessler"` (Kessler pseudo-likelihood).
`optim.method`	the `method` for `optim`.
`lower, upper`	bounds on the variables for the `"Brent"` or `"L-BFGS-B"` method.
`object`	an object inheriting from class `"fitsde"`.
`parm`	a specification of which parameters are to be given confidence intervals, either a vector of names (example `parm='theta1'`). If missing, all parameters are considered.
`level`	the confidence level required.
`...`	potentially further arguments to pass to `optim`.

Details

The function `fitsde` returns a pseudo-likelihood estimators of the drift and diffusion parameters in 1-dim stochastic differential equation. The `optim` optimizer is used to find the maximum of the negative log pseudo-likelihood. An approximate covariance matrix for the parameters is obtained by inverting the Hessian matrix at the optimum.

The `pmle` of pseudo-likelihood can be one among:`"euler"`: Euler pseudo-likelihood), `"ozaki"`: Ozaki pseudo-likelihood, `"shoji"`: Shoji pseudo-likelihood, and `"kessler"`: Kessler pseudo-likelihood.

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

Value

`fitsde` returns an object inheriting from `class` `"fitsde"`.

Author(s)

A.C. Guidoum, K. Boukhetala.

References

Kessler, M. (1997). Estimation of an ergodic diffusion from discrete observations. *Scand. J. Statist.*, 24, 211-229.

Iacus, S.M. (2008). *Simulation and inference for stochastic differential equations: with R examples*. Springer-Verlag, New York.

Iacus, S.M. (2009). sde: Simulation and Inference for Stochastic Differential Equations. R *package version 2.0.10*.

Iacus, S.M. and all. (2014). The yuima Project: A Computational Framework for Simulation and Inference of Stochastic Differential Equations. *Journal of Statistical Software*, **57**(4).

Ozaki, T. (1992). A bridge between nonlinear time series models and nonlinear stochastic dynamical systems: A local linearization approach. *Statistica Sinica*, 2, 25-83.

Shoji, L., Ozaki, T. (1998). Estimation for nonlinear stochastic differential equations by a local linearization method. *Stochastic Analysis and Applications*, 16, 733-752.

Dacunha, D.C. and Florens, D.Z. (1986). Estimation of the Coefficients of a Diffusion from Discrete Observations. *Stochastics*. 19, 263–284.

Dohnal, G. (1987). On estimating the diffusion coefficient. *J. Appl.Prob.*, 24, 105–114.

Genon, V.C. (1990). Maximum constrast estimation for diffusion processes from discrete observation. *Statistics*, 21, 99–116.

Nicolau, J. (2004). Introduction to the estimation of stochastic differential equations based on discrete observations. *Autumn School and International Conference, Stochastic Finance*.

Ait-Sahalia, Y. (1999). Transition densities for interest rate and other nonlinear diffusions. *The Journal of Finance*, 54, 1361–1395.

Ait-Sahalia, Y. (2002). Maximum likelihood estimation of discretely sampled diffusions: a closed-form approximation approach. *Econometrica*. 70, 223–262.

B.L.S. Prakasa Rao. (1999). *Statistical Inference for Diffusion Type Processes*. Arnold, London and Oxford University press, New York.

Kutoyants, Y.A. (2004). *Statistical Inference for Ergodic Diffusion Processes*. Springer, London.

See Also

`dcEuler`, `dcElerian`, `dcOzaki`, `dcShoji`, `dcKessler` and `dcSim` for approximated conditional law of a diffusion process. `gmm` estimator of the generalized method of moments by Hansen, and `HPloglik` these functions are useful to calculate approximated maximum likelihood estimators when the transition density of the process is not known, in package "sde".

`qmle` in package "yuima" calculate quasi-likelihood and ML estimator of least squares estimator.

Examples

```
#####  Example 1:

## Modele GBM (BS)
## dX(t) = theta1 * X(t) * dt + theta2 * x * dW(t)
## Simulation of data
set.seed(1234)

X <- GBM(N =1000,theta=4,sigma=1)
## Estimation: true theta=c(4,1)
fx <- expression(theta[1]*x)
gx <- expression(theta[2]*x)

fres <- fitsde(data=X,drift=fx,diffusion=gx,start = list(theta1=1,theta2=1),
```

```
                 lower=c(0,0))
fres
summary(fres)
coef(fres)
logLik(fres)
AIC(fres)
BIC(fres)
vcov(fres)
confint(fres,level=0.95)
```

`fptsde1d` *Approximate densities and random generation for first passage time in 1-D SDE*

Description

Kernel density and random generation for first-passage-time (f.p.t) in 1-dim stochastic differential equations.

Usage

```
fptsde1d(object, ...)
dfptsde1d(object, ...)

## Default S3 method:
fptsde1d(object, boundary, ...)
## S3 method for class 'fptsde1d'
summary(object, digits=NULL, ...)
## S3 method for class 'fptsde1d'
mean(x, ...)
## S3 method for class 'fptsde1d'
Median(x, ...)
## S3 method for class 'fptsde1d'
Mode(x, ...)
## S3 method for class 'fptsde1d'
quantile(x, ...)
## S3 method for class 'fptsde1d'
kurtosis(x, ...)
## S3 method for class 'fptsde1d'
skewness(x, ...)
## S3 method for class 'fptsde1d'
min(x, ...)
## S3 method for class 'fptsde1d'
max(x, ...)
## S3 method for class 'fptsde1d'
moment(x, ...)
## S3 method for class 'fptsde1d'
```

```
cv(x, ...)

## Default S3 method:
dfptsde1d(object, ...)
## S3 method for class 'dfptsde1d'
plot(x, hist=FALSE, ...)
```

Arguments

`object`	an object inheriting from class `snssde1d` for `fptsde1d`, and `fptsde1d` for `dfptsde1d`.
`boundary`	an `expression` of a constant or time-dependent boundary.
`x`	an object inheriting from class `dfptsde1d`.
`hist`	if `hist=TRUE` plot histogram. Based on `truehist` function.
`digits`	integer, used for number formatting.
`...`	potentially further arguments for (non-default) methods, such as `density` for `dfptsde1d`.

Details

The function `fptsde1d` returns a random variable $\tau_{(X(t),S(t))}$ "first passage time", is defined as :

$$\tau_{(X(t),S(t))} = \{t \geq 0; X_t \geq S(t)\}, \quad if \quad X(t_0) < S(t_0)$$

$$\tau_{(X(t),S(t))} = \{t \geq 0; X_t \leq S(t)\}, \quad if \quad X(t_0) > S(t_0)$$

And `dfptsde1d` returns a kernel density approximation for $\tau_{(X(t),S(t))}$ "first passage time". with $S(t)$ is through a continuous boundary (barrier).

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

Value

`dfptsde1d()`	gives the density estimate of fpt.
`fptsde1d()`	generates random of fpt.

Author(s)

A.C. Guidoum, K. Boukhetala.

References

Argyrakisa, P. and G.H. Weiss (2006). A first-passage time problem for many random walkers. *Physica A*. **363**, 343–347.

Aytug H., G. J. Koehler (2000). New stopping criterion for genetic algorithms. *European Journal of Operational Research*, **126**, 662–674.

Boukhetala, K. (1996) Modelling and simulation of a dispersion pollutant with attractive centre. ed by Computational Mechanics Publications, Southampton ,U.K and Computational Mechanics Inc, Boston, USA, 245–252.

Boukhetala, K. (1998a). Estimation of the first passage time distribution for a simulated diffusion process. *Maghreb Math.Rev*, **7**(1), 1–25.

Boukhetala, K. (1998b). Kernel density of the exit time in a simulated diffusion. *les Annales Maghrebines De L ingenieur*, **12**, 587–589.

Ding, M. and G. Rangarajan. (2004). First Passage Time Problem: A Fokker-Planck Approach. *New Directions in Statistical Physics*. ed by L. T. Wille. Springer. 31–46.

Roman, R.P., Serrano, J. J., Torres, F. (2008). First-passage-time location function: Application to determine first-passage-time densities in diffusion processes. *Computational Statistics and Data Analysis*. **52**, 4132–4146.

Roman, R.P., Serrano, J. J., Torres, F. (2012). An R package for an efficient approximation of first-passage-time densities for diffusion processes based on the FPTL function. *Applied Mathematics and Computation*, **218**, 8408–8428.

Gardiner, C. W. (1997). *Handbook of Stochastic Methods*. Springer-Verlag, New York.

See Also

`fptsde2d` and `fptsde3d` simulation fpt for 2 and 3-dim SDE.

`FPTL` for computes values of the first passage time location (FPTL) function, and `Approx.fpt.density` for approximate first-passage-time (f.p.t.) density in package "fptdApprox".

`GQD.TIpassage` for compute the First Passage Time Density of a GQD With Time Inhomogeneous Coefficients in package "DiffusionRgqd".

Examples

```
## Example 1: Ito SDE
## dX(t) = -4*X(t) *dt + 0.5*dW(t)
## S(t) = 0 (constant boundary)
set.seed(1234)

# SDE 1d
f <- expression( -4*x )
g <- expression( 0.5 )
mod <- snssde1d(drift=f,diffusion=g,x0=2,M=1000)

# boundary
St <- expression(0)

# random
out <- fptsde1d(mod, boundary=St)
out
summary(out)
# density approximate
den <- dfptsde1d(out)
den
plot(den)

## Example 2: Stratonovich SDE
## dX(t) = 0.5*X(t)*t *dt + sqrt(1+X(t)^2) o dW(t)
## S(t) = -0.5*sqrt(t) + exp(t^2) (time-dependent boundary)
```

```
set.seed(1234)

# SDE 1d
f <- expression( 0.5*x*t )
g <- expression( sqrt(1+x^2) )
mod2 <- snssde1d(drift=f,diffusion=g,x0=2,M=1000,type="srt")

# boundary
St <- expression(-0.5*sqrt(t)+exp(t^2))

# random
out2 <- fptsde1d(mod2,boundary=St)
out2
summary(out2)
# density approximate
plot(dfptsde1d(out2,bw='ucv'))


## Example 3: fptsde1d vs fptdApproximate
## Not run:
f <- expression( -0.5*x+0.5*5 )
g <- expression( 1 )
St <- expression(5+0.25*sin(2*pi*t))
mod <- snssde1d(drift=f,diffusion=g,boundary=St,x0=3,T=10,N=10^4,M =10000)
mod

# random
out3 <- fptsde1d(mod,boundary=St)
out3
summary(out3)
# density approximate:
library("fptdApprox")
# Under `fptdApprox':
# Define the diffusion process and give its transitional density:
OU <- diffproc(c("alpha*x + beta","sigma^2",
"dnorm((x-(y*exp(alpha*(t-s)) - beta*(1 - exp(alpha*(t-s)))/alpha))/
(sigma*sqrt((exp(2*alpha*(t-s)) - 1)/(2*alpha))),0,1)/
(sigma*sqrt((exp(2*alpha*(t-s)) - 1)/(2*alpha)))",
"pnorm(x, y*exp(alpha*(t-s)) - beta*(1 - exp(alpha*(t-s)))/alpha,
sigma*sqrt((exp(2*alpha*(t-s)) - 1)/(2*alpha)))"))
# Approximate the first passgage time density for OU, starting in X_0 = 3
# passing through 5+0.25*sin(2*pi*t) on the time interval [0,10]:
res <- Approx.fpt.density(OU, 0, 10, 3,"5+0.25*sin(2*pi*t)", list(alpha=-0.5,beta=0.5*5,sigma=1))

##
plot(dfptsde1d(out3,bw='ucv'),main = 'fptsde1d vs fptdApproximate')
lines(res$y~res$x, type = 'l',lwd=2)
legend('topright', lty = c('solid', 'dashed'), col = c(1, 2),
       legend = c('fptdApproximate', 'fptsde1d'), lwd = 2, bty = 'n')


## End(Not run)
```

fptsde2d *Approximate densities and random generation for first passage time in 2-D SDE's*

Description

Kernel density and random generation for first-passage-time (f.p.t) in 2-dim stochastic differential equations.

Usage

```
fptsde2d(object, ...)
dfptsde2d(object, ...)

## Default S3 method:
fptsde2d(object, boundary, ...)
## S3 method for class 'fptsde2d'
summary(object, digits=NULL, ...)
## S3 method for class 'fptsde2d'
mean(x, ...)
## S3 method for class 'fptsde2d'
Median(x, ...)
## S3 method for class 'fptsde2d'
Mode(x, ...)
## S3 method for class 'fptsde2d'
quantile(x, ...)
## S3 method for class 'fptsde2d'
kurtosis(x, ...)
## S3 method for class 'fptsde2d'
skewness(x, ...)
## S3 method for class 'fptsde2d'
min(x, ...)
## S3 method for class 'fptsde2d'
max(x, ...)
## S3 method for class 'fptsde2d'
moment(x, ...)
## S3 method for class 'fptsde2d'
cv(x, ...)

## Default S3 method:
dfptsde2d(object, pdf=c("Joint","Marginal"), ...)
## S3 method for class 'dfptsde2d'
plot(x,display=c("persp","rgl","image","contour"),
                         hist=FALSE, ...)
```

Arguments

object	an object inheriting from class `snssde2d` for `fptsde2d`, and `fptsde2d` for `dfptsde2d`.

boundary	an expression of a constant or time-dependent boundary.
pdf	probability density function Joint or Marginal.
x	an object inheriting from class fptsde2d.
digits	integer, used for number formatting.
display	display plots.
hist	if hist=TRUE plot histogram. Based on truehist function.
...	potentially further arguments for (non-default) methods. arguments to be passed to methods, such as density for marginal density and kde2d fro joint density.

Details

The function fptsde1d returns a random variable $(\tau_{(X(t),S(t))},\tau_{(Y(t),S(t))})$ "first passage time", is defined as :

$$\tau_{(X(t),S(t))} = \{t \geq 0; X_t \geq S(t)\}, \quad if \quad X(t_0) < S(t_0)$$
$$\tau_{(Y(t),S(t))} = \{t \geq 0; Y_t \geq S(t)\}, \quad if \quad Y(t_0) < S(t_0)$$

and:

$$\tau_{(X(t),S(t))} = \{t \geq 0; X_t \leq S(t)\}, \quad if \quad X(t_0) > S(t_0)$$
$$\tau_{(Y(t),S(t))} = \{t \geq 0; Y_t \leq S(t)\}, \quad if \quad Y(t_0) > S(t_0)$$

And dfptsde2d returns a kernel density approximation for $(\tau_{(X(t),S(t))},\tau_{(Y(t),S(t))})$ "first passage time". with $S(t)$ is through a continuous boundary (barrier).

An overview of this package, see browseVignettes('Sim.DiffProc') for more informations.

Value

dfptsde2d()	gives the kernel density approximation for fpt.
fptsde2d()	generates random of fpt.

Author(s)

A.C. Guidoum, K. Boukhetala.

References

Argyrakisa, P. and G.H. Weiss (2006). A first-passage time problem for many random walkers. *Physica A*. **363**, 343–347.

Aytug H., G. J. Koehler (2000). New stopping criterion for genetic algorithms. *European Journal of Operational Research*, **126**, 662–674.

Boukhetala, K. (1996) Modelling and simulation of a dispersion pollutant with attractive centre. ed by Computational Mechanics Publications, Southampton ,U.K and Computational Mechanics Inc, Boston, USA, 245–252.

Boukhetala, K. (1998a). Estimation of the first passage time distribution for a simulated diffusion process. *Maghreb Math.Rev*, **7**(1), 1–25.

Boukhetala, K. (1998b). Kernel density of the exit time in a simulated diffusion. *les Annales Maghrebines De L ingenieur*, **12**, 587–589.

Ding, M. and G. Rangarajan. (2004). First Passage Time Problem: A Fokker-Planck Approach. *New Directions in Statistical Physics*. ed by L. T. Wille. Springer. 31–46.

Roman, R.P., Serrano, J. J., Torres, F. (2008). First-passage-time location function: Application to determine first-passage-time densities in diffusion processes. *Computational Statistics and Data Analysis*. **52**, 4132–4146.

Roman, R.P., Serrano, J. J., Torres, F. (2012). An R package for an efficient approximation of first-passage-time densities for diffusion processes based on the FPTL function. *Applied Mathematics and Computation*, **218**, 8408–8428.

Gardiner, C. W. (1997). *Handbook of Stochastic Methods*. Springer-Verlag, New York.

See Also

`fptsde1d` for simulation fpt in sde 1-dim. `fptsde3d` for simulation fpt in sde 3-dim.

`FPTL` for computes values of the first passage time location (FPTL) function, and `Approx.fpt.density` for approximate first-passage-time (f.p.t.) density in package "fptdApprox".

`GQD.TIpassage` for compute the First Passage Time Density of a GQD With Time Inhomogeneous Coefficients in package "DiffusionRgqd".

Examples

```
## dX(t) = 5*(-1-Y(t))*X(t) * dt + 0.5 * dW1(t)
## dY(t) = 5*(-1-X(t))*Y(t) * dt + 0.5 * dW2(t)
## x0 = 2, y0 = -2, and barrier -3+5*t.
## W1(t) and W2(t) two independent Brownian motion
set.seed(1234)

# SDE's 2d
fx <- expression(5*(-1-y)*x , 5*(-1-x)*y)
gx <- expression(0.5 , 0.5)
mod2d <- snssde2d(drift=fx,diffusion=gx,x0=c(2,-2),M=100)

# boundary

St <- expression(-1+5*t)

# random fpt

out <- fptsde2d(mod2d,boundary=St)
out
summary(out)

# Marginal density

denM <- dfptsde2d(out,pdf="M")
denM
plot(denM)

# Joint density

denJ <- dfptsde2d(out,pdf="J",n=200,lims=c(0.28,0.4,0.04,0.13))
```

```
denJ
plot(denJ)
plot(denJ,display="image")
plot(denJ,display="image",drawpoints=TRUE,cex=0.5,pch=19,col.pt='green')
plot(denJ,display="contour")
plot(denJ,display="contour",color.palette=colorRampPalette(c('white','green','blue','red')))
```

`fptsde3d` *Approximate densities and random generation for first passage time in 3-D SDE's*

Description

Kernel density and random generation for first-passage-time (f.p.t) in 3-dim stochastic differential equations.

Usage

```
fptsde3d(object, ...)
dfptsde3d(object, ...)

## Default S3 method:
fptsde3d(object, boundary, ...)
## S3 method for class 'fptsde3d'
summary(object, digits=NULL, ...)
## S3 method for class 'fptsde3d'
mean(x, ...)
## S3 method for class 'fptsde3d'
Median(x, ...)
## S3 method for class 'fptsde3d'
Mode(x, ...)
## S3 method for class 'fptsde3d'
quantile(x, ...)
## S3 method for class 'fptsde3d'
kurtosis(x, ...)
## S3 method for class 'fptsde3d'
skewness(x, ...)
## S3 method for class 'fptsde3d'
min(x, ...)
## S3 method for class 'fptsde3d'
max(x, ...)
## S3 method for class 'fptsde3d'
moment(x, ...)
## S3 method for class 'fptsde3d'
cv(x, ...)

## Default S3 method:
dfptsde3d(object, pdf=c("Joint","Marginal"), ...)
```

```
## S3 method for class 'dfptsde3d'
plot(x,display="rgl",hist=FALSE, ...)
```

Arguments

`object`	an object inheriting from class `snssde3d` for `fptsde3d`, and `fptsde3d` for `dfptsde3d`.
`boundary`	an `expression` of a constant or time-dependent boundary.
`pdf`	probability density function `Joint` or `Marginal`.
`x`	an object inheriting from class `dfptsde3d`.
`digits`	integer, used for number formatting.
`display`	display plots.
`hist`	if `hist=TRUE` plot histogram. Based on `truehist` function.
`...`	potentially arguments to be passed to methods, such as `density` for marginal density and `sm.density` for joint density.

Details

The function `fptsde3d` returns a random variable $(\tau_{(X(t),S(t))}, \tau_{(Y(t),S(t))}, \tau_{(Z(t),S(t))})$ "first passage time", is defined as :

$$\tau_{(X(t),S(t))} = \{t \geq 0; X_t \geq S(t)\}, \quad if \quad X(t_0) < S(t_0)$$

$$\tau_{(Y(t),S(t))} = \{t \geq 0; Y_t \geq S(t)\}, \quad if \quad Y(t_0) < S(t_0)$$

$$\tau_{(Z(t),S(t))} = \{t \geq 0; Z_t \geq S(t)\}, \quad if \quad Z(t_0) < S(t_0)$$

and:

$$\tau_{(X(t),S(t))} = \{t \geq 0; X_t \leq S(t)\}, \quad if \quad X(t_0) > S(t_0)$$

$$\tau_{(Y(t),S(t))} = \{t \geq 0; Y_t \leq S(t)\}, \quad if \quad Y(t_0) > S(t_0)$$

$$\tau_{(Z(t),S(t))} = \{t \geq 0; Z_t \leq S(t)\}, \quad if \quad Z(t_0) > S(t_0)$$

And `dfptsde3d` returns a marginal kernel density approximation for $(\tau_{(X(t),S(t))}, \tau_{(Y(t),S(t))}, \tau_{(Z(t),S(t))})$ "first passage time". with $S(t)$ is through a continuous boundary (barrier).

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

Value

`dfptsde3d()`	gives the marginal kernel density approximation for fpt.
`fptsde3d()`	generates random of fpt.

Author(s)

A.C. Guidoum, K. Boukhetala.

References

Argyrakisa, P. and G.H. Weiss (2006). A first-passage time problem for many random walkers. *Physica A*. **363**, 343–347.

Aytug H., G. J. Koehler (2000). New stopping criterion for genetic algorithms. *European Journal of Operational Research*, **126**, 662–674.

Boukhetala, K. (1996) Modelling and simulation of a dispersion pollutant with attractive centre. ed by Computational Mechanics Publications, Southampton ,U.K and Computational Mechanics Inc, Boston, USA, 245–252.

Boukhetala, K. (1998a). Estimation of the first passage time distribution for a simulated diffusion process. *Maghreb Math.Rev*, **7**(1), 1–25.

Boukhetala, K. (1998b). Kernel density of the exit time in a simulated diffusion. *les Annales Maghrebines De L ingenieur*, **12**, 587–589.

Ding, M. and G. Rangarajan. (2004). First Passage Time Problem: A Fokker-Planck Approach. *New Directions in Statistical Physics*. ed by L. T. Wille. Springer. 31–46.

Roman, R.P., Serrano, J. J., Torres, F. (2008). First-passage-time location function: Application to determine first-passage-time densities in diffusion processes. *Computational Statistics and Data Analysis*. **52**, 4132–4146.

Roman, R.P., Serrano, J. J., Torres, F. (2012). An R package for an efficient approximation of first-passage-time densities for diffusion processes based on the FPTL function. *Applied Mathematics and Computation*, **218**, 8408–8428.

Gardiner, C. W. (1997). *Handbook of Stochastic Methods*. Springer-Verlag, New York.

See Also

`fptsde1d` for simulation fpt in sde 1-dim. `fptsde2d` for simulation fpt in sde 2-dim.

`FPTL` for computes values of the first passage time location (FPTL) function, and `Approx.fpt.density` for approximate first-passage-time (f.p.t.) density in package "fptdApprox".

`GQD.TIpassage` for compute the First Passage Time Density of a GQD With Time Inhomogeneous Coefficients in package "DiffusionRgqd".

Examples

```
## dX(t) = 4*(-1-X(t))*Y(t) dt + 0.2 * dW1(t)
## dY(t) = 4*(1-Y(t)) *X(t) dt + 0.2 * dW2(t)
## dZ(t) = 4*(1-Z(t)) *Y(t) dt + 0.2 * dW3(t)
## x0 = 0, y0 = -2, z0 = 0, and barrier -3+5*t.
## W1(t), W2(t) and W3(t) three independent Brownian motion
set.seed(1234)

# SDE's 3d

fx <- expression(4*(-1-x)*y, 4*(1-y)*x, 4*(1-z)*y)
gx <- rep(expression(0.2),3)
mod3d <- snssde3d(drift=fx,diffusion=gx,M=500)

# boundary
```

```
St <- expression(-3+5*t)

# random

out <- fptsde3d(mod3d,boundary=St)
out
summary(out)

# Marginal density

denM <- dfptsde3d(out,pdf="M")
denM
plot(denM)

# Multiple isosurfaces
## Not run:
denJ <- dfptsde3d(out,pdf="J")
denJ
plot(denJ,display="rgl")

## End(Not run)
```

HWV *Hull-White/Vasicek, Ornstein-Uhlenbeck process*

Description

The (S3) generic function for simulation of Hull-White/Vasicek or gaussian diffusion models, and Ornstein-Uhlenbeck process.

Usage

```
HWV(N, ...)
OU(N, ...)

## Default S3 method:
HWV(N = 100, M = 1, x0 = 2, t0 = 0, T = 1, Dt = NULL, mu = 4, theta = 1,
   sigma = 0.1, ...)
## Default S3 method:
OU(N =100,M=1,x0=2,t0=0,T=1,Dt = NULL,mu=4,sigma=0.2, ...)
```

Arguments

N	number of simulation steps.
M	number of trajectories.
x0	initial value of the process at time t_0.
t0	initial time.
T	final time.

Dt	time step of the simulation (discretization). If it is missing a default $\Delta t = \frac{T-t_0}{N}$.
mu	parameter of the HWV and OU; see details.
theta	parameter of the HWV; see details.
sigma	the volatility of the HWV and OU.
...	potentially further arguments for (non-default) methods.

Details

The function HWV returns a trajectory of the **Hull-White/Vasicek process** starting at x_0 at time t_0; i.e., the diffusion process solution of stochastic differential equation:

$$dX_t = \mu(\theta - X_t)dt + \sigma dW_t$$

The function OU returns a trajectory of the **Ornstein-Uhlenbeck** starting at x_0 at time t_0; i.e., the diffusion process solution of stochastic differential equation:

$$dX_t = -\mu X_t dt + \sigma dW_t$$

Constraints: $\mu, \sigma > 0$.

Please note that the process is stationary only if $\mu > 0$.

Value

X	an visible ts object.

Author(s)

A.C. Guidoum, K. Boukhetala.

References

Vasicek, O. (1977). An Equilibrium Characterization of the Term Structure. *Journal of Financial Economics*, 5, 177–188.

See Also

rcOU and rsOU for conditional and stationary law of Vasicek process are available in "sde".

Examples

```
## Hull-White/Vasicek Models
## dX(t) = 4 * (2.5 - X(t)) * dt + 1 *dW(t), X0=10
set.seed(1234)

X <- HWV(N=1000,M=10,mu = 4, theta = 2.5,sigma = 1,x0=10)
plot(X,plot.type="single")
lines(as.vector(time(X)),rowMeans(X),col="red")
```

```
## Ornstein-Uhlenbeck Process
## dX(t) = -4 * X(t) * dt + 1 *dW(t) , X0=2
set.seed(1234)

X <- OU(N=1000,M=10,mu = 4,sigma = 1,x0=10)
plot(X,plot.type="single")
lines(as.vector(time(X)),rowMeans(X),col="red")
```

Irates *Monthly Interest Rates*

Description

monthly observations from 1946–12 to 1991–02

number of observations : 531

observation : country

country : United–States

Usage

```
data(Irates)
```

Format

A time serie containing :

r1 interest rate for a maturity of 1 months (% per year).

r2 interest rate for a maturity of 2 months (% per year).

r3 interest rate for a maturity of 3 months (% per year).

r5 interest rate for a maturity of 5 months (% per year).

r6 interest rate for a maturity of 6 months (% per year).

r11 interest rate for a maturity of 11 months (% per year).

r12 interest rate for a maturity of 12 months (% per year).

r36 interest rate for a maturity of 36 months (% per year).

r60 interest rate for a maturity of 60 months (% per year).

r120 interest rate for a maturity of 120 months (% per year).

Source

McCulloch, J.H. and Kwon, H.C. (1993). U.S. term structure data, 1947–1991, Ohio State Working Paper 93–6, Ohio State University, Columbus

These datasets `Irates` are in package "Ecdat".

References

Croissant, Y. (2014). Ecdat: Data sets for econometrics. R package version 0.2–5.

Examples

```
data(Irates)
rates <- Irates[,"r1"]
rates <- window(rates, start=1964.471, end=1989.333)

## CKLS modele vs CIR modele
## CKLS :  dX(t) = (theta1+theta2* X(t))* dt + theta3 * X(t)^theta4 * dW(t)

fx <- expression(theta[1]+theta[2]*x)
gx <- expression(theta[3]*x^theta[4])
fitmod <- fitsde(rates,drift=fx,diffusion=gx,pmle="euler",start = list(theta1=1,theta2=1,
                  theta3=1,theta4=1),optim.method = "L-BFGS-B")
theta <- coef(fitmod)

N <- length(rates)
res <- snssde1d(drift=fx,diffusion=gx,M=1000,t0=time(rates)[1],T=time(rates)[N],
                Dt=deltat(rates),x0=rates[1],N=N)

plot(res,type="n",ylim=c(0,35))
lines(rates,col=2,lwd=2)
lines(time(res),apply(res$X,1,mean),col=3,lwd=2)
lines(time(res),apply(res$X,1,bconfint,level=0.95)[1,],col=4,lwd=2)
lines(time(res),apply(res$X,1,bconfint,level=0.95)[2,],col=4,lwd=2)
legend("topleft",c("real data","mean path",
  paste("bound of", 95," confidence")),inset = .01,
  col=2:4,lwd=2,cex=0.8)
```

MCM.sde — *Parallel Monte-Carlo Methods for SDE's*

Description

Generate R Monte-Carlo (version parallel) replicates of a statistic applied to SDE's (1,2 and 3 dim) for the two cases Ito and Stratonovich interpretations.

Usage

```
MCM.sde(model, ...)

## Default S3 method:
MCM.sde(model, statistic, R = 100, time, exact = NULL,
        names = NULL, level = 0.95, parallel = c("no", "multicore", "snow"),
        ncpus = getOption("ncpus", 1L), cl = NULL, ...)

## S3 method for class 'MCM.sde'
plot(x,index = 1,type=c("all","hist","qqplot","boxplot","CI"), ...)
```

Arguments

`model`	an object from class snssde1d, snssde2d and snssde3d.
`statistic`	a function which when applied to model returns a vector containing the statistic(s) of interest.
`R`	the number of Monte-Carlo replicates. Usually this will be a single positive integer "R > 1".
`time`	the time when estimating the statistic(s) of interesttime between `t0` and `T`. The default `time = T`.
`exact`	a named list giving the exact statistic(s) if it exists otherwise `exact = NULL`.
`names`	named the statistic(s) of interest. The default `names=c("mu1","mu2",...)`.
`level`	the confidence level(s) of the required interval(s).
`parallel`	the type of parallel operation to be used (if any). The default `parallel = "no"`.
`ncpus`	integer: number of processes to be used in parallel operation: typically one would chose this to the number of available CPUs.
`cl`	an optional parallel or snow cluster for use if `parallel = "snow"`.
`x`	an object inheriting from class `"MCM.sde"`.
`index`	the index of the variable of interest within the output of `"MCM.sde"`.
`type`	the type of plot of the Monte-Carlo estimation of the variable of interest. The default `type = "all"`.
`...`	potentially further arguments for (non-default) methods.

Details

We have here developed Monte-Carlo methods whose essence is the use of repeated experiments to evaluate a statistic(s) of interest in SDE's. For example estimation of moments as: mean, variance, covariance (and other as median, mode, quantile,...). With the standard error and the confidence interval for these estimators.

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

Value

The returned value is an object of class `"MCM.sde"`, containing the following components:

`mod`	The SDE's used (class: snssde1d, snssde2d and snssde3d).
`dim`	Dimension of the model.
`call`	The original call to `"MCM.sde"`.
`Fn`	The function statistic as passed to `"MCM.sde"`.
`ech`	A matrix with `sum(R)` column each of which is a Monte-Carlo replicate of the result of calling statistic.
`time`	The time when estimating the statistic(s) of interest.
`name`	named of statistic(s) of interest.
`MC`	Table contains simulation results of statistic(s) of interest: Estimate, Bias (if exact available), Std.Error and Confidence interval.

Note

When parallel = "multicore" is used are not available on Windows, parallel = "snow" is primarily intended to be used on multi-core Windows machine where parallel = "multicore" is not available. For more details see Q.E.McCallum and S.Weston (2011).

Author(s)

A.C. Guidoum, K. Boukhetala.

References

Guidoum AC, Boukhetala K (2020). "Performing Parallel Monte Carlo and Moment Equations Methods for Itô and Stratonovich Stochastic Differential Systems: R Package Sim.DiffProc". *Journal of Statistical Software*, **96**(2), 1–82. doi:10.18637/jss.v096.i02

Paul Glasserman (2003). *Monte Carlo Methods in Financial Engineering*. Springer-Verlag New York.

Jun S. Liu (2004). *Monte Carlo Strategies in Scientific Computing*. Springer-Verlag New York.

Christian Robert and George Casella (2010). *Introducing Monte Carlo Methods with R*. Springer-Verlag New York.

Nick T. Thomopoulos (2013). *Essentials of Monte Carlo Simulation: Statistical Methods for Building Simulation Models*. Springer-Verlag New York.

Q. Ethan McCallum and Stephen Weston (2011). *Parallel R*. O'Reilly Media, Inc.

See Also

MEM.sde moment equations methods for SDE's.

Examples

```
## Example 1 : (1 dim)
## dX(t) = 3*(1-X(t)) dt + 0.5 * dW(t), X(0)=5, t in [0,10]
## set the model 1d
f <- expression(3*(1-x));g <- expression(0.5)
mod1d <-  snssde1d(drift=f,diffusion=g,x0=5,T=10,M=50)

## function of the statistic(s) of interest.
sde.fun1d <- function(data, i){
  d <- data[i, ]
  return(c(mean(d),Mode(d),var(d)))
}

mc.sde1d = MCM.sde(model=mod1d,statistic=sde.fun1d,R=100,exact=list(Me=1,Mo=1,Va=0.5^2/6),
                 names=c("Me(10)","Mo(10)","Va(10)"))
mc.sde1d
plot(mc.sde1d,index=1)
plot(mc.sde1d,index=2)
plot(mc.sde1d,index=3)

## Example 2 : with Parallel computing
```

```
## Not run:
mod1d <-  snssde1d(drift=f,diffusion=g,x0=5,T=10,M=1000)
## On Windows or Unix
mc.sde1d = MCM.sde(model=mod1d,statistic=sde.fun1d,R=1000,exact=list(Me=1,Mo=1,Va=0.5^2/6),
       names=c("Me(10)","Mo(10)","Va(10)"),parallel="snow",ncpus=parallel::detectCores())
mc.sde1d
## On Unix only
mc.sde1d = MCM.sde(model=mod1d,statistic=sde.fun1d,R=1000,exact=list(Me=1,Mo=1,Va=0.5^2/6),
       names=c("Me(10)","Mo(10)","Va(10)"),parallel="multicore",ncpus=parallel::detectCores())
mc.sde1d

## End(Not run)

## Example 3: (2 dim)
## dX(t) = 1/mu*(theta-X(t)) dt + sqrt(sigma) * dW1(t),
## dY(t) = X(t) dt + 0 * dW2(t)
## Not run:
## Set the model 2d
mu=0.75;sigma=0.1;theta=2
x0=0;y0=0;init=c(x=0,y=0)
f <- expression(1/mu*(theta-x), x)
g <- expression(sqrt(sigma),0)
OUI <- snssde2d(drift=f,diffusion=g,M=1000,Dt=0.01,x0=init)

## function of the statistic(s) of interest.
sde.fun2d <- function(data, i){
  d <- data[i,]
  return(c(mean(d$x),mean(d$y),var(d$x),var(d$y),cov(d$x,d$y)))
}
## Monte-Carlo at time = 5
mc.sde2d_a = MCM.sde(model=OUI,statistic=sde.fun2d,R=100,time=5,
                  parallel="snow",ncpus=parallel::detectCores())
mc.sde2d_a
## Monte-Carlo at time = 10
mc.sde2d_b = MCM.sde(model=OUI,statistic=sde.fun2d,R=100,time=10,
                  parallel="snow",ncpus=parallel::detectCores())
mc.sde2d_b

## Compared with exact values at time 5 and 10
E_x <- function(t) theta+(x0-theta)*exp(-t/mu)
V_x <- function(t) 0.5*sigma*mu *(1-exp(-2*(t/mu)))
E_y <- function(t) y0+theta*t+(x0-theta)*mu*(1-exp(-t/mu))
V_y <- function(t) sigma*mu^3*((t/mu)-2*(1-exp(-t/mu))+0.5*(1-exp(-2*(t/mu))))
cov_xy <- function(t) 0.5*sigma*mu^2 *(1-2*exp(-t/mu)+exp(-2*(t/mu)))

## at time=5
mc.sde2d_a = MCM.sde(model=OUI,statistic=sde.fun2d,R=100,time=5,
    exact=list(m1=E_x(5),m2=E_y(5),S1=V_x(5),S2=V_y(5),C12=cov_xy(5)),
parallel="snow",ncpus=parallel::detectCores())
mc.sde2d_a
plot(mc.sde2d_a,index=1)
plot(mc.sde2d_a,index=2)
## at time=10
```

```
mc.sde2d_b = MCM.sde(model=OUI,statistic=sde.fun2d,R=100,time=10,
    exact=list(m1=E_x(10),m2=E_y(10),S1=V_x(10),S2=V_y(10),C12=cov_xy(10)),
parallel="snow",ncpus=parallel::detectCores())
mc.sde2d_b
plot(mc.sde2d_b,index=1)
plot(mc.sde2d_b,index=2)

## End(Not run)

## Example 4: (3 dim)
## dX(t) = sigma*(Y(t)-X(t)) dt + 0.1 * dW1(t)
## dY(t) = (rho*X(t)-Y(t)-X(t)*Z(t)) dt + 0.1 * dW2(t)
## dZ(t) = (X(t)*Y(t)-bet*Z(t)) dt + 0.1 * dW3(t)
## W1(t), W2(t) and W3(t) are three correlated Brownian motions with Sigma
## Not run:
## Set the model 3d
sigma=10;rho=28; bet=8/3
f <- expression(sigma*(y-x),rho*x-y-x*z,x*y-bet*z)
g <- expression(0.1,0.1,0.1)
# correlation matrix
Sigma <-matrix(c(1,0.3,0.5,0.3,1,0.2,0.5,0.2,1),nrow=3,ncol=3)
mod3d <- snssde3d(x0=rep(0,3),drift=f,diffusion=g,M=1000,Dt=0.01,corr=Sigma)

## function of the statistic(s) of interest.
sde.fun3d <- function(data, i){
  d <- data[i,]
  return(c(mean(d$x),mean(d$y),mean(d$z)))
}
## Monte-Carlo at time = 10
mc.sde3d = MCM.sde(mod3d,statistic=sde.fun3d,R=100,parallel="snow",ncpus=parallel::detectCores())
mc.sde3d

## End(Not run)
```

MEM.sde *Moment Equations Methods for SDE's*

Description

Calculate and numerical approximation of moment equations (Symbolic ODE's of means and variances-covariance) at any time for SDE's (1,2 and 3 dim) for the two cases Ito and Stratonovich interpretations.

Usage

```
MEM.sde(drift, diffusion, ...)

## Default S3 method:
MEM.sde(drift, diffusion, corr = NULL, type = c("ito", "str"), solve = FALSE,
```

```
        parms = NULL, init = NULL, time = NULL, ...)

## S3 method for class 'MEM.sde'
summary(object, at , ...)
```

Arguments

`drift`	drift coefficient: an `expression` 1-dim(t,x), 2-dim(t,x,y) or 3-dim(t,x,y,z).
`diffusion`	diffusion coefficient: an `expression` 1-dim(t,x), 2-dim(t,x,y) or 3-dim(t,x,y,z).
`corr`	the correlation coefficient '|corr|<=1' of W1(t) and W2(t) (2d) must be an expression length equal 1. And for 3d (W1(t),W2(t),W3(t)) an expressions length equal 3. See examples.
`type`	type of process `"ito"` or `"Stratonovich"`; the default `type="ito"`.
`solve`	if `solve=TRUE` solves a system of ordinary differential equations.
`parms`	parameters passed to `drift` and `diffusion`.
`init`	the initial (state) values for the ODE system. for 1-dim (m=x0,S=0), 2-dim (m1=x0,m2=y0,S1=0,S2=0,C12=0) and for 3-dim (m1=x0,m2=y0,m3=z0,S1=0,S2=0,S3=0,C12=0,C13= see examples.
`time`	time sequence (vector) for which output is wanted; the first value of time must be the initial time.
`object, at`	an object inheriting from class `"MEM.sde"` and summaries at any time `at`.
`...`	potentially arguments to be passed to methods, such as `ode` for solver for ODE's.

Details

The stochastic transition is approximated by the moment equations, and the numerical treatment is required to solve these equations from above with given initial conditions.

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

Value

Symbolic ODE's of means and variances-covariance. If `solve=TRUE` approximate the moment of SDE's at any time.

Author(s)

A.C. Guidoum, K. Boukhetala.

References

Guidoum AC, Boukhetala K (2020). "Performing Parallel Monte Carlo and Moment Equations Methods for Itô and Stratonovich Stochastic Differential Systems: R Package Sim.DiffProc". *Journal of Statistical Software*, **96**(2), 1–82. doi:10.18637/jss.v096.i02

Rodriguez R, Tuckwell H (2000). *A dynamical system for the approximate moments of nonlinear stochastic models of spiking neurons and networks.* Mathematical and Computer Modelling, 31(4), 175–180.

Alibrandi U, Ricciardi G (2012). *Stochastic Methods in Nonlinear Structural Dynamics*, 3–60. Springer Vienna, Vienna. ISBN 978-3-7091-1306-6.

See Also

MCM.sde Monte-Carlo methods for SDE's.

Examples

```
library(deSolve)
## Example 1: 1-dim
## dX(t) = mu * X(t) * dt + sigma * X(t) * dW(t)
## Symbolic ODE's of mean and variance
f <- expression(mu*x)
g <- expression(sigma*x)
res1 <- MEM.sde(drift=f,diffusion=g,type="ito")
res2 <- MEM.sde(drift=f,diffusion=g,type="str")
res1
res2
## numerical approximation of mean and variance
para <- c(mu=2,sigma=0.5)
t    <- seq(0,1,by=0.001)
init <- c(m=1,S=0)
res1 <- MEM.sde(drift=f,diffusion=g,solve=TRUE,init=init,parms=para,time=t)
res1
matplot.0D(res1$sol.ode,main="Mean and Variance of X(t), type Ito")
plot(res1$sol.ode,select=c("m","S"))
## approximation at time = 0.75
summary(res1,at=0.75)

##
res2 <- MEM.sde(drift=f,diffusion=g,solve=TRUE,init=init,parms=para,time=t,type="str")
res2
matplot.0D(res2$sol.ode,main="Mean and Variance of X(t), type Stratonovich")
plot(res2$sol.ode,select=c("m","S"))
## approximation at time = 0.75
summary(res2,at=0.75)

## Comparison:

plot(res1$sol.ode, res2$sol.ode,ylab = c("m(t)"),select="m",xlab = "Time",
     col = c("red", "blue"))
plot(res1$sol.ode, res2$sol.ode,ylab = c("S(t)"),select="S",xlab = "Time",
     col = c("red", "blue"))

## Example2: 2-dim
## dX(t) = 1/mu*(theta-X(t)) dt + sqrt(sigma) * dW1(t),
## dY(t) = X(t) dt + 0 * dW2(t)
## Not run:
para=c(mu=0.75,sigma=0.1,theta=2)
init=c(m1=0,m2=0,S1=0,S2=0,C12=0)
t <- seq(0,10,by=0.001)
f <- expression(1/mu*(theta-x), x)
g <- expression(sqrt(sigma),0)
res2d <- MEM.sde(drift=f,diffusion=g,solve=TRUE,init=init,parms=para,time=t)
res2d
```

```
## Exact moment

mu=0.75;sigma=0.1;theta=2;x0=0;y0=0
E_x <- function(t) theta+(x0-theta)*exp(-t/mu)
V_x <- function(t) 0.5*sigma*mu *(1-exp(-2*(t/mu)))
E_y <- function(t) y0+theta*t+(x0-theta)*mu*(1-exp(-t/mu))
V_y <- function(t) sigma*mu^3*((t/mu)-2*(1-exp(-t/mu))+0.5*(1-exp(-2*(t/mu))))
cov_xy <- function(t) 0.5*sigma*mu^2 *(1-2*exp(-t/mu)+exp(-2*(t/mu)))

##
summary(res2d,at=5)
E_x(5);E_y(5);V_x(5);V_y(5);cov_xy(5)

matplot.0D(res2d$sol.ode,select=c("m1"))
curve(E_x,add=TRUE,col="red")

## plot

plot(res2d$sol.ode)
matplot.0D(res2d$sol.ode,select=c("S1","S2","C12"))
plot(res2d$sol.ode[,"m1"], res2d$sol.ode[,"m2"], xlab = "m1(t)",
  ylab = "m2(t)", type = "l",lwd = 2)
hist(res2d$sol.ode,select=c("m1","m2"), col = c("darkblue", "red", "orange", "black"))

## Example3: 2-dim with correlation
## Heston model
## dX(t) = mu*X(t) dt + sqrt(Y(t))*X(t) * dW1(t),
## dY(t) = lambda*(theta-Y(t)) dt + sigma*sqrt(Y(t)) * dW2(t)
## with E(dw1dw2)=rho


f <- expression( mu*x, lambda*(theta-y) )
g <- expression( sqrt(y)*x, sigma*sqrt(y) )
RHO  <- expression(rho)
res2d <- MEM.sde(drift=f,diffusion=g,corr=RHO)
res2d

## Numerical approximation
RHO <- expression(0.5)
para=c(mu=1,lambda=3,theta=0.5,sigma=0.1)
ini=c(m1=10,m2=2,S1=0,S2=0,C12=0)
res2d = MEM.sde(drift=f,diffusion=g,solve=TRUE,parms=para,init=ini,time=seq(0,1,by=0.01))
res2d

matplot.0D(res2d$sol.ode,select=c("m1","m2"))
matplot.0D(res2d$sol.ode,select=c("S1","S2","C12"))

## Example4: 3-dim
## dX(t) = sigma*(Y(t)-X(t)) dt + 0.1 * dW1(t)
## dY(t) = (rho*X(t)-Y(t)-X(t)*Z(t)) dt + 0.1 * dW2(t)
## dZ(t) = (X(t)*Y(t)-bet*Z(t)) dt + 0.1 * dW3(t)
## with E(dw1dw2)=rho1, E(dw1dw3)=rho2 and E(dw2dw3)=rho3
```

```
f <- expression(sigma*(y-x),rho*x-y-x*z,x*y-bet*z)
g <- expression(0.1,0.1,0.1)
RHO <- expression(rho1,rho2,rho3)
## Symbolic moments equations
res3d = MEM.sde(drift=f,diffusion=g,corr=RHO)
res3d

## Numerical approximation
RHO <- expression(0.5,0.2,-0.7)
para=c(sigma=10,rho=28,bet=8/3)
ini=c(m1=1,m2=1,m3=1,S1=0,S2=0,S3=0,C12=0,C13=0,C23=0)
res3d = MEM.sde(drift=f,diffusion=g,solve=T,parms=para,init=ini,time=seq(0,1,by=0.01))
res3d

summary(res3d,at=0.25)
summary(res3d,at=0.50)
summary(res3d,at=0.75)

plot(res3d$sol.ode)
matplot.0D(res3d$sol.ode,select=c("m1","m2","m3"))
matplot.0D(res3d$sol.ode,select=c("S1","S2","S3"))
matplot.0D(res3d$sol.ode,select=c("C12","C13","C23"))

plot3D(res3d$sol.ode[,"m1"], res3d$sol.ode[,"m2"],res3d$sol.ode[,"m3"], xlab = "m1(t)",
  ylab = "m2(t)",zlab="m3(t)", type = "l",lwd = 2,box=F)

## End(Not run)
```

`moment` *Monte-Carlo statistics of SDE's*

Description

Generic function for compute the kurtosis, skewness, median, mode and coefficient of variation (relative variability), moment and confidence interval of class "sde".

Usage

```
## Default S3 method:
bconfint(x, level = 0.95, ...)
## Default S3 method:
kurtosis(x, ...)
## Default S3 method:
moment(x, order = 1,center = TRUE, ...)
## Default S3 method:
cv(x, ...)
## Default S3 method:
```

```
skewness(x, ...)
## Default S3 method:
Median(x, ...)
## Default S3 method:
Mode(x, ...)
```

Arguments

x	an object inheriting from class "sde".
order	order of moment.
center	if TRUE is a central moment.
level	the confidence level required.
...	potentially further arguments for (non-default) methods.

Author(s)

A.C. Guidoum, K. Boukhetala.

Examples

```
## Example 1:
## dX(t) = 2*(3-X(t)) *dt + dW(t)
set.seed(1234)

f <- expression( 2*(3-x) )
g <- expression( 1 )
mod <- snssde1d(drift=f,diffusion=g,M=10000,T=5)
## Monte-Carlo statistics of 5000 trajectory of X(t) at final time T of 'mod'
summary(mod)
kurtosis(mod)
skewness(mod)
mean(mod)
Median(mod)
Mode(mod)
moment(mod,order=4)
cv(mod)
bconfint(mod,level = 0.95) ## of mean
```

---

plot2d *Plotting for Class SDE*

Description

Generic function for plotting.

Usage

```
## Default S3 method:
plot2d(x, ...)
## Default S3 method:
lines2d(x, ...)
## Default S3 method:
points2d(x, ...)
## Default S3 method:
plot3D(x, display = c("persp","rgl"), ...)
```

Arguments

x	an object inheriting from class snssde2d, snssde3d, bridgesde2d and bridgesde3d.
display	"persp" perspective or "rgl" plots.
...	other graphics parameters, see par in package "graphics", scatterplot3d in package "scatterplot3d" and plot3d in package "rgl".

Details

The 2 and 3-dim plot of class sde.

Author(s)

A.C. Guidoum, K. Boukhetala.

Examples

```
## Example 1:
set.seed(1234)

fx <- rep(expression(0),2)
gx <- rep(expression(1),2)

res <- snssde2d(drift=fx,diffusion=gx,N=5000)
plot2d(res,type="l")
```

---

rsde1d *Approximate transitional densities and random generation for 1-D SDE*

Description

Transition density and random generation for `X(t-s) | X(s)=x0` of the 1-dim SDE.

Usage

```
rsde1d(object, ...)
dsde1d(object, ...)

## Default S3 method:
rsde1d(object, at, ...)

## Default S3 method:
dsde1d(object, at, ...)
## S3 method for class 'dsde1d'
plot(x,hist=FALSE, ...)
```

Arguments

object	an object inheriting from class snssde1d and bridgesde1d.
at	time between s=t0 and t=T. The default at = T.
x	an object inheriting from class dsde1d.
hist	if hist=TRUE plot histogram. Based on truehist function.
...	potentially arguments to be passed to methods, such as density for kernel density.

Details

The function rsde1d returns a M random variable $x_{t=at}$ realize at time $t = at$ defined by :

$$x_{t=at} = \{t \geq 0; x = X_{t=at}\}$$

And dsde1d returns a transition density approximation for X(t-s) | X(s)=x0. with $t = at$ is a fixed time between t0 and T.

An overview of this package, see browseVignettes('Sim.DiffProc') for more informations.

Value

dsde1d()	gives the transition density estimate of X(t-s) | X(s)=x0.
rsde1d()	generates random of X(t-s) | X(s)=x0.

Author(s)

A.C. Guidoum, K. Boukhetala.

See Also

density Kernel density estimation in "stats" package.

kde Kernel density estimate for 1- to 6-dimensional data in "ks" package.

sm.density Nonparametric density estimation in one, two or three dimensions in "sm" package.

rng random number generators in "yuima" package.

dcSim Pedersen's simulated transition density in "sde" package.

rcBS, rcCIR, rcOU and rsOU in package "sde".

dcBS, dcCIR, dcOU and dsOU in package "sde".

GQD.density Generate the transition density of a scalar generalized quadratic diffusion in "DiffusionRgqd" package.

Examples

```
## Example 1:
## dX(t) = (-2*(X(t)<=0)+2*(X(t)>=0)) *dt + 0.5 * dW(t)
set.seed(1234)

f <- expression(-2*(x<=0)+2*(x>=0))
g <- expression(0.5)
res1 <- snssde1d(drift=f,diffusion=g,M=5000)
x <- rsde1d(res1, at = 1)
summary(x)
dens1 <-  dsde1d(res1, at = 1)
dens1
plot(dens1,main="Transition density of X(t=1)|X(s=0)=0") # kernel estimated
plot(dens1,hist=TRUE) # histogramme

## Example 2:
## Transition density of standard Brownian motion W(t) at time = 0.5
set.seed(1234)

f <- expression(0)
g <- expression(1)
res2 <- snssde1d(drift=f,diffusion=g,M=5000)
plot(dsde1d(res2, at = 0.5),dens=function(x) dnorm(x,0,sqrt(0.5)))
plot(dsde1d(res2, at = 0.5),dens=function(x) dnorm(x,0,sqrt(0.5)),hist=TRUE)

## Example 3: Transition density of Brownian motion W(t) in [0,1]

## Not run:
for (i in seq(res2$t0,res2$T,by=res2$Dt)){
plot(dsde1d(res2, at = i),main=paste0('Transition Density \n t = ',i))
}

## End(Not run)

## Example 4:
## Transition density of bridge Brownian motion W(t) at time = 0.25 and 0.75
set.seed(1234)
## Not run:
f <- expression(0)
g <- expression(1)
Bd <- bridgesde1d(drift=f,diffusion=g,M=5000)
Bd
plot(dsde1d(Bd, at = 0.25))         ## Transition Density at time=0.25
plot(dsde1d(Bd, at = 0.75),add=TRUE)## Transition Density at time=0.75
```

```
## End(Not run)
```

---

rsde2d — *Approximate transitional densities and random generation for 2-D SDE's*

Description

Transition density and random generation for the joint and marginal of `(X(t-s),Y(t-s) | X(s)=x0,Y(s)=y0)` of the SDE's 2-d.

Usage

```
rsde2d(object, ...)
dsde2d(object, ...)

## Default S3 method:
rsde2d(object, at, ...)

## Default S3 method:
dsde2d(object, pdf=c("Joint","Marginal"), at, ...)
## S3 method for class 'dsde2d'
plot(x,display=c("persp","rgl","image","contour"),hist=FALSE,...)
```

Arguments

`object`	an object inheriting from class `snssde2d` and `bridgesde2d`.
`at`	time between `s=t0` and `t=T`. The default `at = T`.
`pdf`	probability density function `Joint` or `Marginal`.
`x`	an object inheriting from class `dsde2d`.
`display`	display plots.
`hist`	if `hist=TRUE` plot histogram. Based on `truehist` function.
`...`	potentially potentially arguments to be passed to methods, such as `density` for marginal density and `kde2d` fro joint density.

Details

The function `rsde2d` returns a `M` random variable $x_{t=at}, y_{t=at}$ realize at time $t = at$.

And `dsde2d` returns a bivariate density approximation for `(X(t-s),Y(t-s) | X(s)=x0,Y(s)=y0)`. with $t = at$ is a fixed time between `t0` and `T`.

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

Value

dsde2d() gives the bivariate density approximation for `(X(t-s),Y(t-s) | X(s)=x0,Y(s)=y0)`.

rsde2d() generates random of the couple `(X(t-s),Y(t-s) | X(s)=x0,Y(s)=y0)`.

Author(s)

A.C. Guidoum, K. Boukhetala.

See Also

kde2d Two-dimensional kernel density estimation in "MASS" package.

kde Kernel density estimate for 1- to 6-dimensional data in "ks" package.

sm.density Nonparametric density estimation in one, two or three dimensions in "sm" package.

rng random number generators in "yuima" package.

BiGQD.density Generate the transition density of a bivariate generalized quadratic diffusion model (2D GQD) in "DiffusionRgqd" package.

Examples

```
## Example:1
set.seed(1234)

# SDE's 2d
fx <- expression(3*(2-y),2*x)
gx <- expression(1,y)
mod2d <- snssde2d(drift=fx,diffusion=gx,x0=c(1,2),M=1000)

# random
r2d <- rsde2d(mod2d,at=0.5)
summary(r2d)

# Marginal density

denM <- dsde2d(mod2d,pdf="M", at=0.5)
denM
plot(denM)

# Joint density
denJ <- dsde2d(mod2d,pdf="J",n=200, at= 0.5,lims=c(-3,4,0,6))
denJ
plot(denJ)
plot(denJ,display="contour")

## Example 2: Bivariate Transition Density of 2 Brownian motion (W1(t),W2(t)) in [0,1]

## Not run:
B2d <- snssde2d(drift=rep(expression(0),2),diffusion=rep(expression(1),2),
       M=10000)
for (i in seq(B2d$Dt,B2d$T,by=B2d$Dt)){
plot(dsde2d(B2d, at = i,lims=c(-3,3,-3,3),n=100),
```

```
    display="contour",main=paste0('Transition Density \n t = ',i))
}

## End(Not run)

## Example 3:

## Not run:
fx <- expression(4*(-1-x)*y , 4*(1-y)*x )
gx <- expression(0.25*y,0.2*x)
mod2d1 <- snssde2d(drift=fx,diffusion=gx,x0=c(x0=1,y0=-1),
      M=5000,type="str")

# Marginal transition density
for (i in seq(mod2d1$Dt,mod2d1$T,by=mod2d1$Dt)){
plot(dsde2d(mod2d1,pdf="M", at = i),main=
      paste0('Marginal Transition Density \n t = ',i))
}

# Bivariate transition density
for (i in seq(mod2d1$Dt,mod2d1$T,by=mod2d1$Dt)){
plot(dsde2d(mod2d1, at = i,lims=c(-1,2,-1,1),n=100),
    display="contour",main=paste0('Transition Density \n t = ',i))
}

## End(Not run)

## Example 4: Bivariate Transition Density of 2 bridge Brownian motion (W1(t),W2(t)) in [0,1]

## Not run:
B2d <- bridgesde2d(drift=rep(expression(0),2),
  diffusion=rep(expression(1),2),M=5000)
for (i in seq(0.01,0.99,by=B2d$Dt)){
plot(dsde2d(B2d, at = i,lims=c(-3,3,-3,3),
 n=100),display="contour",main=
 paste0('Transition Density \n t = ',i))
}

## End(Not run)

## Example 5: Bivariate Transition Density of bridge
## Ornstein-Uhlenbeck process and its integral in [0,5]
## dX(t) = 4*(-1-X(t)) dt + 0.2 dW1(t)
## dY(t) = X(t) dt + 0 dW2(t)
## x01 = 0 , y01 = 0
## x02 = 0, y02 = 0
## Not run:
fx <- expression(4*(-1-x) , x)
gx <- expression(0.2 , 0)
OUI <- bridgesde2d(drift=fx,diffusion=gx,Dt=0.005,M=1000)
for (i in seq(0.01,4.99,by=OUI$Dt)){
plot(dsde2d(OUI, at = i,lims=c(-1.2,0.2,-2.5,0.2),n=100),
 display="contour",main=paste0('Transition Density \n t = ',i))
```

```
}

## End(Not run)
```

rsde3d — *Approximate transitional densities and random generation for 3-D SDE's*

Description

Transition density and random generation for the joint and marginal of `(X(t-s),Y(t-s),Z(t-s) | X(s)=x0,Y(s)=y0,Z(s)=z0)` of the SDE's 3-d.

Usage

```
rsde3d(object, ...)
dsde3d(object, ...)

## Default S3 method:
rsde3d(object, at, ...)

## Default S3 method:
dsde3d(object, pdf=c("Joint","Marginal"), at, ...)
## S3 method for class 'dsde3d'
plot(x,display="rgl",hist=FALSE,...)
```

Arguments

`object`	an object inheriting from class `snssde3d` and `bridgesde3d`.
`at`	time between `s=t0` and `t=T`. The default `at = T`.
`pdf`	probability density function `Joint` or `Marginal`.
`x`	an object inheriting from class `dsde3d`.
`display`	display plots.
`hist`	if `hist=TRUE` plot histogram. Based on `truehist` function.
`...`	potentially arguments to be passed to methods, such as `density` for marginal density and `sm.density` for joint density.

Details

The function `rsde3d` returns a `M` random variable $x_{t=at}, y_{t=at}, z_{t=at}$ realize at time $t = at$.

And `dsde3d` returns a trivariate kernel density approximation for `(X(t-s),Y(t-s),Z(t-s) | X(s)=x0,Y(s)=y0,Z(s)=z0)`. with $t = at$ is a fixed time between `t0` and `T`.

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

Value

`dsde3d()` gives the trivariate density approximation `(X(t-s),Y(t-s),Z(t-s) | X(s)=x0,Y(s)=y0,Z(s)=z0)`.

`rsde3d()` generates random of the `(X(t-s),Y(t-s),Z(t-s) | X(s)=x0,Y(s)=y0,Z(s)=z0)`.

Author(s)

A.C. Guidoum, K. Boukhetala.

See Also

`kde` Kernel density estimate for 1- to 6-dimensional data in "ks" package.

`sm.density` Nonparametric density estimation in one, two or three dimensions in "sm" package.

`kde3d` Compute a three dimension kernel density estimate in "misc3d" package.

`rng` random number generators in "yuima" package.

`rcBS`, `rcCIR`, `rcOU` and `rsOU` in package "sde".

Examples

```
## Example 1: Ito sde
## dX(t) = (2*(Y(t)>0)-2*(Z(t)<=0)) dt + 0.2 * dW1(t)
## dY(t) = -2*Y(t) dt + 0.2 * dW2(t)
## dZ(t) = -2*Z(t) dt + 0.2 * dW3(t)
## W1(t), W2(t) and W3(t) three independent Brownian motion
set.seed(1234)
fx <- expression(2*(y>0)-2*(z<=0) , -2*y, -2*z)
gx <- rep(expression(0.2),3)
mod3d1 <- snssde3d(x0=c(0,2,-2),drift=fx,diffusion=gx,M=1000,Dt=0.003)

# random at t= 0.75
r3d1 <- rsde3d(mod3d1,at=0.75)
summary(r3d1)

# Marginal transition density at t=0.75, t0=0

denM <- dsde3d(mod3d1,pdf="M",at=0.75)
denM
plot(denM)

# for Joint transition density at t=0.75;t0=0
# Multiple isosurfaces
## Not run:
denJ <- dsde3d(mod3d1,pdf="J", at= 0.75)
denJ
plot(denJ,display="rgl")

## End(Not run)

## Example 2: Stratonovich sde
## dX(t) = Y(t)* dt + X(t) o dW1(t)
## dY(t) = (4*( 1-X(t)^2 )* Y(t) - X(t))* dt + 0.2 o dW2(t)
```

```
## dZ(t) = (4*( 1-X(t)^2 )* Z(t) - X(t))* dt + 0.2 o dW3(t)
set.seed(1234)

fx <- expression( y , (4*( 1-x^2 )* y - x), (4*( 1-x^2 )* z - x))
gx <- expression( x , 0.2, 0.2)
mod3d2 <- snssde3d(drift=fx,diffusion=gx,M=1000,type="str")

# random
r3d2 <- rsde3d(mod3d2)
summary(r3d2)

# Marginal transition density at t=1, t0=0

denM <- dsde3d(mod3d2,pdf="M")
denM
plot(denM)

# for Joint transition density at t=1;t0=0
# Multiple isosurfaces
## Not run:
denJ <- dsde3d(mod3d2,pdf="J")
denJ
plot(denJ,display="rgl")

## End(Not run)

## Example 3: Tivariate Transition Density of 3 Brownian motion (W1(t),W2(t),W3(t)) in [0,1]

## Not run:
B3d <- snssde3d(drift=rep(expression(0),3),diffusion=rep(expression(1),3),M=500)
for (i in seq(B3d$Dt,B3d$T,by=B3d$Dt)){
plot(dsde3d(B3d, at = i,pdf="J"),box=F,main=paste0('Transition Density t = ',i))
}

## End(Not run)
```

---

snssde1d *Simulation of 1-D Stochastic Differential Equation*

Description

The (S3) generic function snssde1d of simulation of solution to 1-dim stochastic differential equation of Itô or Stratonovich type, with different methods.

Usage

```
snssde1d(N, ...)
## Default S3 method:
snssde1d(N = 1000, M = 1, x0 = 0, t0 = 0, T = 1, Dt,
```

```
    drift, diffusion, alpha = 0.5, mu = 0.5, type = c("ito", "str"),
    method = c("euler", "milstein", "predcorr", "smilstein", "taylor",
    "heun", "rk1", "rk2", "rk3"), ...)

## S3 method for class 'snssde1d'
summary(object, at ,digits=NULL, ...)
## S3 method for class 'snssde1d'
time(x, ...)
## S3 method for class 'snssde1d'
mean(x, at, ...)
## S3 method for class 'snssde1d'
Median(x, at, ...)
## S3 method for class 'snssde1d'
Mode(x, at, ...)
## S3 method for class 'snssde1d'
quantile(x, at, ...)
## S3 method for class 'snssde1d'
kurtosis(x, at, ...)
## S3 method for class 'snssde1d'
min(x, at, ...)
## S3 method for class 'snssde1d'
max(x, at, ...)
## S3 method for class 'snssde1d'
skewness(x, at, ...)
## S3 method for class 'snssde1d'
moment(x, at,  ...)
## S3 method for class 'snssde1d'
cv(x, at,  ...)
## S3 method for class 'snssde1d'
bconfint(x, at,  ...)

## S3 method for class 'snssde1d'
plot(x, ...)
## S3 method for class 'snssde1d'
lines(x, ...)
## S3 method for class 'snssde1d'
points(x, ...)
```

Arguments

N	number of simulation steps.
M	number of trajectories (Monte-Carlo).
x0	initial value of the process at time t0.
t0	initial time.
T	ending time.
Dt	time step of the simulation (discretization). If it is missing a default $\Delta t = \frac{T-t_0}{N}$.

`drift`	drift coefficient: an `expression` of two variables `t` and `x`.
`diffusion`	diffusion coefficient: an `expression` of two variables `t` and `x`.
`alpha, mu`	weight of the predictor-corrector scheme; the default `alpha = 0.5` and `mu = 0.5`.
`type`	if `type="ito"` simulation sde of Itô type, else `type="str"` simulation sde of Stratonovich type; the default `type="ito"`.
`method`	numerical methods of simulation, the default `method = "euler"`.
`x, object`	an object inheriting from class `"snssde1d"`.
`at`	time between `t0` and `T`. Monte-Carlo statistics of the solution X_t at time `at`. The default `at = T`.
`digits`	integer, used for number formatting.
`...`	potentially further arguments for (non-default) methods.

Details

The function `snssde1d` returns a `ts` x of length `N+1`; i.e. solution of the sde of Ito or Stratonovich types; If `Dt` is not specified, then the best discretization $\Delta t = \frac{T-t_0}{N}$.

The Ito stochastic differential equation is:

$$dX(t) = a(t, X(t))dt + b(t, X(t))dW(t)$$

Stratonovich sde :

$$dX(t) = a(t, X(t))dt + b(t, X(t)) \circ dW(t)$$

The methods of approximation are classified according to their different properties. Mainly two criteria of optimality are used in the literature: the strong and the weak (orders of) convergence. The `method` of simulation can be one among: `Euler-Maruyama Order 0.5`, `Milstein Order 1`, `Milstein Second-Order`, `Predictor-Corrector method`, `Itô-Taylor Order 1.5`, `Heun Order 2` and `Runge-Kutta Order 1, 2 and 3`.

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

Value

`snssde1d` returns an object inheriting from `class "snssde1d"`.

`X`	an invisible `ts` object.
`drift`	drift coefficient.
`diffusion`	diffusion coefficient.
`type`	type of sde.
`method`	the numerical method used.

Author(s)

A.C. Guidoum, K. Boukhetala.

References

Guidoum AC, Boukhetala K (2020). "Performing Parallel Monte Carlo and Moment Equations Methods for Itô and Stratonovich Stochastic Differential Systems: R Package Sim.DiffProc". *Journal of Statistical Software*, **96**(2), 1–82. doi:10.18637/jss.v096.i02

Friedman, A. (1975). *Stochastic differential equations and applications*. Volume 1, ACADEMIC PRESS.

Henderson, D. and Plaschko,P. (2006). *Stochastic differential equations in science and engineering*. World Scientific.

Allen, E. (2007). *Modeling with Ito stochastic differential equations*. Springer-Verlag.

Jedrzejewski, F. (2009). *Modeles aleatoires et physique probabiliste*. Springer-Verlag.

Iacus, S.M. (2008). *Simulation and inference for stochastic differential equations: with R examples*. Springer-Verlag, New York.

Kloeden, P.E, and Platen, E. (1989). A survey of numerical methods for stochastic differential equations. *Stochastic Hydrology and Hydraulics*, **3**, 155–178.

Kloeden, P.E, and Platen, E. (1991a). Relations between multiple ito and stratonovich integrals. *Stochastic Analysis and Applications*, **9**(3), 311–321.

Kloeden, P.E, and Platen, E. (1991b). Stratonovich and ito stochastic taylor expansions. *Mathematische Nachrichten*, **151**, 33–50.

Kloeden, P.E, and Platen, E. (1995). *Numerical Solution of Stochastic Differential Equations*. Springer-Verlag, New York.

Oksendal, B. (2000). *Stochastic Differential Equations: An Introduction with Applications*. 5th edn. Springer-Verlag, Berlin.

Platen, E. (1980). Weak convergence of approximations of ito integral equations. *Z Angew Math Mech*. **60**, 609–614.

Platen, E. and Bruti-Liberati, N. (2010). *Numerical Solution of Stochastic Differential Equations with Jumps in Finance.* Springer-Verlag, New York

Saito, Y, and Mitsui, T. (1993). Simulation of Stochastic Differential Equations. *The Annals of the Institute of Statistical Mathematics*, **3**, 419–432.

See Also

`snssde2d` and `snssde3d` for 2 and 3-dim sde.

`sde.sim` in package "sde".

`simulate` in package "yuima".

Examples

```
## Example 1: Ito sde
## dX(t) = 2*(3-X(t)) dt + 2*X(t) dW(t)
set.seed(1234)

f <- expression(2*(3-x) )
g <- expression(1)
mod1 <- snssde1d(drift=f,diffusion=g,M=4000,x0=10,Dt=0.01)
```

```
mod1
summary(mod1)
## Not run:
plot(mod1)
lines(time(mod1),apply(mod1$X,1,mean),col=2,lwd=2)
lines(time(mod1),apply(mod1$X,1,bconfint,level=0.95)[1,],col=4,lwd=2)
lines(time(mod1),apply(mod1$X,1,bconfint,level=0.95)[2,],col=4,lwd=2)
legend("topright",c("mean path",paste("bound of", 95," percent confidence")),
       inset = .01,col=c(2,4),lwd=2,cex=0.8)

## End(Not run)
## Example 2: Stratonovich sde
## dX(t) = ((2-X(t))/(2-t)) dt + X(t) o dW(t)
set.seed(1234)

f <- expression((2-x)/(2-t))
g <- expression(x)
mod2 <- snssde1d(type="str",drift=f,diffusion=g,M=4000,x0=1, method="milstein")
mod2
summary(mod2,at = 0.25)
summary(mod2,at = 1)
## Not run:
plot(mod2)
lines(time(mod2),apply(mod2$X,1,mean),col=2,lwd=2)
lines(time(mod2),apply(mod2$X,1,bconfint,level=0.95)[1,],col=4,lwd=2)
lines(time(mod2),apply(mod2$X,1,bconfint,level=0.95)[2,],col=4,lwd=2)
legend("topleft",c("mean path",paste("bound of", 95," percent confidence")),
       inset = .01,col=c(2,4),lwd=2,cex=0.8)

## End(Not run)
```

---

snssde2d *Simulation of 2-D Stochastic Differential Equation*

Description

The (S3) generic function snssde2d of simulation of solutions to 2-dim stochastic differential equations of Itô or Stratonovich type, with different methods.

Usage

```
snssde2d(N, ...)
## Default S3 method:
snssde2d(N = 1000, M = 1, x0 = c(0,0),t0 = 0, T = 1, Dt,
   drift, diffusion, corr = NULL, type = c("ito", "str"), alpha = 0.5, mu = 0.5,
   method = c("euler", "milstein", "predcorr", "smilstein", "taylor",
   "heun", "rk1", "rk2","rk3"), ...)

## S3 method for class 'snssde2d'
```

```
summary(object, at, digits=NULL,...)
## S3 method for class 'snssde2d'
time(x, ...)
## S3 method for class 'snssde2d'
mean(x, at, ...)
## S3 method for class 'snssde2d'
Median(x, at, ...)
## S3 method for class 'snssde2d'
Mode(x, at, ...)
## S3 method for class 'snssde2d'
quantile(x, at, ...)
## S3 method for class 'snssde2d'
kurtosis(x, at, ...)
## S3 method for class 'snssde2d'
skewness(x, at, ...)
## S3 method for class 'snssde2d'
min(x, at, ...)
## S3 method for class 'snssde2d'
max(x, at, ...)
## S3 method for class 'snssde2d'
moment(x, at, ...)
## S3 method for class 'snssde2d'
cv(x, at, ...)
## S3 method for class 'snssde2d'
bconfint(x, at, ...)

## S3 method for class 'snssde2d'
plot(x, ...)
## S3 method for class 'snssde2d'
lines(x, ...)
## S3 method for class 'snssde2d'
points(x, ...)
## S3 method for class 'snssde2d'
plot2d(x, ...)
## S3 method for class 'snssde2d'
lines2d(x, ...)
## S3 method for class 'snssde2d'
points2d(x, ...)
```

Arguments

N	number of simulation steps.
M	number of trajectories (Monte-Carlo).
x0	initial values x0=(x,y) of the process X_t and Y_t at time t0.
t0	initial time.
T	ending time.
Dt	time step of the simulation (discretization). If it is missing a default $\Delta t = \frac{T-t_0}{N}$.

`drift`	drift coefficient: an `expression` of three variables `t`, `x` and `y` for process X_t and Y_t.
`diffusion`	diffusion coefficient: an `expression` of three variables `t`, `x` and `y` for process X_t and Y_t.
`corr`	the correlation structure of two Brownian motions W1(t) and W2(t); must be a real symmetric positive-definite square matrix of dimension 2.
`type`	if `type="ito"` simulation sde of Itô type, else `type="str"` simulation sde of Stratonovich type; the default `type="ito"`.
`alpha, mu`	weight of the predictor-corrector scheme; the default `alpha = 0.5` and `mu = 0.5`.
`method`	numerical methods of simulation, the default `method = "euler"`.
`x, object`	an object inheriting from class `"snssde2d"`.
`at`	time between `t0` and `T`. Monte-Carlo statistics of the solutions (X_t, Y_t) at time `at`. The default `at = T`.
`digits`	integer, used for number formatting.
`...`	potentially further arguments for (non-default) methods.

Details

The function `snssde2d` returns a `mts` x of length `N+1`; i.e. solution of the 2-dim sde (X_t, Y_t) of Ito or Stratonovich types; If `Dt` is not specified, then the best discretization $\Delta t = \frac{T-t_0}{N}$.

The 2-dim Ito stochastic differential equation is:

$$dX(t) = a(t, X(t), Y(t))dt + b(t, X(t), Y(t))dW_1(t)$$
$$dY(t) = a(t, X(t), Y(t))dt + b(t, X(t), Y(t))dW_2(t)$$

2-dim Stratonovich sde :

$$dX(t) = a(t, X(t), Y(t))dt + b(t, X(t), Y(t)) \circ dW_1(t)$$
$$dY(t) = a(t, X(t), Y(t))dt + b(t, X(t), Y(t)) \circ dW_2(t)$$

$W_1(t), W_2(t)$ are two standard Brownian motion independent if `corr=NULL`.

In the correlation case, currently we can use only the Euler-Maruyama and Milstein scheme.

The methods of approximation are classified according to their different properties. Mainly two criteria of optimality are used in the literature: the strong and the weak (orders of) convergence. The `method` of simulation can be one among: `Euler-Maruyama Order 0.5`, `Milstein Order 1`, `Milstein Second-Order`, `Predictor-Corrector method`, `Itô-Taylor Order 1.5`, `Heun Order 2` and `Runge-Kutta Order 1, 2 and 3`.

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

Value

`snssde2d` returns an object inheriting from `class` `"snssde2d"`.

`X, Y`	an invisible `mts` (2-dim) object (X(t),Y(t)).
`driftx, drifty`	drift coefficient of X(t) and Y(t).
`diffx, diffy`	diffusion coefficient of X(t) and Y(t).
`type`	type of sde.
`method`	the numerical method used.

Author(s)

A.C. Guidoum, K. Boukhetala.

References

Guidoum AC, Boukhetala K (2020). "Performing Parallel Monte Carlo and Moment Equations Methods for Itô and Stratonovich Stochastic Differential Systems: R Package Sim.DiffProc". *Journal of Statistical Software*, **96**(2), 1–82. doi:10.18637/jss.v096.i02

Friedman, A. (1975). *Stochastic differential equations and applications*. Volume 1, ACADEMIC PRESS.

Henderson, D. and Plaschko,P. (2006). *Stochastic differential equations in science and engineering*. World Scientific.

Allen, E. (2007). *Modeling with Ito stochastic differential equations*. Springer-Verlag.

Jedrzejewski, F. (2009). *Modeles aleatoires et physique probabiliste*. Springer-Verlag.

Iacus, S.M. (2008). *Simulation and inference for stochastic differential equations: with R examples*. Springer-Verlag, New York.

Kloeden, P.E, and Platen, E. (1989). A survey of numerical methods for stochastic differential equations. *Stochastic Hydrology and Hydraulics*, **3**, 155–178.

Kloeden, P.E, and Platen, E. (1991a). Relations between multiple ito and stratonovich integrals. *Stochastic Analysis and Applications*, **9**(3), 311–321.

Kloeden, P.E, and Platen, E. (1991b). Stratonovich and ito stochastic taylor expansions. *Mathematische Nachrichten*, **151**, 33–50.

Kloeden, P.E, and Platen, E. (1995). *Numerical Solution of Stochastic Differential Equations*. Springer-Verlag, New York.

Oksendal, B. (2000). *Stochastic Differential Equations: An Introduction with Applications*. 5th edn. Springer-Verlag, Berlin.

Platen, E. (1980). Weak convergence of approximations of ito integral equations. *Z Angew Math Mech*. **60**, 609–614.

Platen, E. and Bruti-Liberati, N. (2010). *Numerical Solution of Stochastic Differential Equations with Jumps in Finance.* Springer-Verlag, New York

Saito, Y, and Mitsui, T. (1993). Simulation of Stochastic Differential Equations. *The Annals of the Institute of Statistical Mathematics*, **3**, 419–432.

See Also

`snssde3d` for 3-dim sde.

`simulate` in package "yuima".

Examples

```
## Example 1: Ito sde
## dX(t) = 4*(-1-X(t))*Y(t) dt + 0.2 dW1(t)
## dY(t) = 4*(1-Y(t))*X(t) dt + 0.2 dW2(t)
set.seed(1234)
```

```
fx <- expression(4*(-1-x)*y , 4*(1-y)*x )
gx <- expression(0.25*y,0.2*x)

mod2d1 <- snssde2d(drift=fx,diffusion=gx,x0=c(x0=1,y0=-1),M=1000)
mod2d1
summary(mod2d1)
##
dev.new()
plot(mod2d1,type="n")
mx <- apply(mod2d1$X,1,mean)
my <- apply(mod2d1$Y,1,mean)
lines(time(mod2d1),mx,col=1)
lines(time(mod2d1),my,col=2)
legend("topright",c(expression(E(X[t])),expression(E(Y[t]))),lty=1,inset = .01,col=c(1,2),cex=0.95)
##
dev.new()
plot2d(mod2d1) ## in plane (O,X,Y)
lines(my~mx,col=2)

## Now W1(t) and W2(t) are correlated.

set.seed(1234)
Sigma <- matrix(c(0.9,0.3,0.3,0.4),nrow=2,ncol=2) # correlation structure
mod2d1 <- snssde2d(drift=fx,diffusion=gx,corr=Sigma,x0=c(x0=1,y0=-1),M=1000)
mod2d1
summary(mod2d1)
##
dev.new()
plot(mod2d1,type="n")
mx <- apply(mod2d1$X,1,mean)
my <- apply(mod2d1$Y,1,mean)
lines(time(mod2d1),mx,col=1)
lines(time(mod2d1),my,col=2)
legend("topright",c(expression(E(X[t])),expression(E(Y[t]))),lty=1,inset = .01,col=c(1,2),cex=0.95)
##
dev.new()
plot2d(mod2d1) ## in plane (O,X,Y)
lines(my~mx,col=2)

## Example 2: Stratonovich sde
## dX(t) = Y(t) dt + 0 o dW1(t)
## dY(t) = (4*(1-X(t)^2)*Y(t) - X(t) ) dt + 0.2 o dW2(t)
set.seed(1234)

fx <- expression( y , (4*( 1-x^2 )* y - x))
gx <- expression( 0 , 0.2)

mod2d2 <- snssde2d(drift=fx,diffusion=gx,type="str",T=100,N=10000)
mod2d2
plot(mod2d2,pos=2)
dev.new()
plot(mod2d2,union = FALSE)
```

```
dev.new()
plot2d(mod2d2,type="n") ## in plane (O,X,Y)
points2d(mod2d2,col=rgb(0,100,0,50,maxColorValue=255), pch=16)
```

---

snssde3d *Simulation of 3-D Stochastic Differential Equation*

Description

The (S3) generic function snssde3d of simulation of solutions to 3-dim stochastic differential equations of Itô or Stratonovich type, with different methods.

Usage

```
snssde3d(N, ...)
## Default S3 method:
snssde3d(N = 1000, M =1, x0=c(0,0,0), t0 = 0, T = 1, Dt,
   drift, diffusion, corr = NULL, alpha = 0.5, mu = 0.5, type = c("ito", "str"),
   method = c("euler", "milstein","predcorr", "smilstein", "taylor",
   "heun", "rk1", "rk2", "rk3"), ...)

## S3 method for class 'snssde3d'
summary(object, at, digits=NULL,...)
## S3 method for class 'snssde3d'
time(x, ...)
## S3 method for class 'snssde3d'
mean(x, at, ...)
## S3 method for class 'snssde3d'
Median(x, at, ...)
## S3 method for class 'snssde3d'
Mode(x, at, ...)
## S3 method for class 'snssde3d'
quantile(x, at, ...)
## S3 method for class 'snssde3d'
kurtosis(x, at, ...)
## S3 method for class 'snssde3d'
skewness(x, at, ...)
## S3 method for class 'snssde3d'
min(x, at, ...)
## S3 method for class 'snssde3d'
max(x, at, ...)
## S3 method for class 'snssde3d'
moment(x, at, ...)
## S3 method for class 'snssde3d'
cv(x, at, ...)
## S3 method for class 'snssde3d'
bconfint(x, at, ...)
```

```
## S3 method for class 'snssde3d'
plot(x, ...)
## S3 method for class 'snssde3d'
lines(x, ...)
## S3 method for class 'snssde3d'
points(x, ...)
## S3 method for class 'snssde3d'
plot3D(x, display = c("persp","rgl"), ...)
```

Arguments

`N`	number of simulation steps.
`M`	number of trajectories.
`x0`	initial value of the process X_t, Y_t and Z_t at time `t0`.
`t0`	initial time.
`T`	ending time.
`Dt`	time step of the simulation (discretization). If it is `missing` a default $\Delta t = \frac{T-t_0}{N}$.
`drift`	drift coefficient: an `expression` of four variables `t`, `x`, `y` and `z` for process X_t, Y_t and Z_t.
`diffusion`	diffusion coefficient: an `expression` of four variables `t`, `x`, `y` and `z` for process X_t, Y_t and Z_t.
`corr`	the correlation structure of three Brownian motions W1(t), W2(t) and W3(t); must be a real symmetric positive-definite square matrix of dimension 3.
`alpha, mu`	weight of the predictor-corrector scheme; the default `alpha = 0.5` and `mu = 0.5`.
`type`	if `type="ito"` simulation sde of Itô type, else `type="str"` simulation sde of Stratonovich type; the default `type="ito"`.
`method`	numerical methods of simulation, the default `method = "euler"`.
`x, object`	an object inheriting from class `"snssde3d"`.
`at`	time between `t0` and `T`. Monte-Carlo statistics of the solutions (X_t, Y_t, Z_t) at time `at`. The default `at = T`.
`digits`	integer, used for number formatting.
`display`	`"persp"` perspective or `"rgl"` plots.
`...`	potentially further arguments for (non-default) methods.

Details

The function `snssde3d` returns a `mts` x of length `N+1`; i.e. solution of the 3-dim sde (X_t, Y_t, Z_t) of Ito or Stratonovich types; If `Dt` is not specified, then the best discretization $\Delta t = \frac{T-t_0}{N}$.

The 3-dim Ito stochastic differential equation is:

$$dX(t) = a(t, X(t), Y(t), Z(t))dt + b(t, X(t), Y(t), Z(t))dW_1(t)$$

$$dY(t) = a(t, X(t), Y(t), Z(t))dt + b(t, X(t), Y(t), Z(t))dW_2(t)$$

$$dZ(t) = a(t, X(t), Y(t), Z(t))dt + b(t, X(t), Y(t), Z(t))dW_3(t)$$

3-dim Stratonovich sde :

$$dX(t) = a(t, X(t), Y(t), Z(t))dt + b(t, X(t), Y(t), Z(t)) \circ dW_1(t)$$

$$dY(t) = a(t, X(t), Y(t), Z(t))dt + b(t, X(t), Y(t), Z(t)) \circ dW_2(t)$$

$$dZ(t) = a(t, X(t), Y(t), Z(t))dt + b(t, X(t), Y(t), Z(t)) \circ dW_3(t)$$

$W_1(t), W_2(t), W_3(t)$ three standard Brownian motion independent if `corr=NULL`.

In the correlation case, currently we can use only the Euler-Maruyama and Milstein scheme.

The methods of approximation are classified according to their different properties. Mainly two criteria of optimality are used in the literature: the strong and the weak (orders of) convergence. The `method` of simulation can be one among: `Euler-Maruyama Order 0.5`, `Milstein Order 1`, `Milstein Second-Order`, `Predictor-Corrector method`, `Itô-Taylor Order 1.5`, `Heun Order 2` and `Runge-Kutta Order 1, 2 and 3`.

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

Value

`snssde3d` returns an object inheriting from `class "snssde3d"`.

`X, Y, Z`	an invisible `mts` (3-dim) object (X(t),Y(t),Z(t)).
`driftx, drifty, driftz`	drift coefficient of X(t), Y(t) and Z(t).
`diffx, diffy, diffz`	diffusion coefficient of X(t), Y(t) and Z(t).
`type`	type of sde.
`method`	the numerical method used.

Author(s)

A.C. Guidoum, K. Boukhetala.

References

Guidoum AC, Boukhetala K (2020). "Performing Parallel Monte Carlo and Moment Equations Methods for Itô and Stratonovich Stochastic Differential Systems: R Package Sim.DiffProc". *Journal of Statistical Software*, **96**(2), 1–82. doi:10.18637/jss.v096.i02

Friedman, A. (1975). *Stochastic differential equations and applications*. Volume 1, ACADEMIC PRESS.

Henderson, D. and Plaschko,P. (2006). *Stochastic differential equations in science and engineering*. World Scientific.

Allen, E. (2007). *Modeling with Ito stochastic differential equations*. Springer-Verlag.

Jedrzejewski, F. (2009). *Modeles aleatoires et physique probabiliste*. Springer-Verlag.

Iacus, S.M. (2008). *Simulation and inference for stochastic differential equations: with R examples*. Springer-Verlag, New York.

Kloeden, P.E, and Platen, E. (1989). A survey of numerical methods for stochastic differential equations. *Stochastic Hydrology and Hydraulics*, **3**, 155–178.

Kloeden, P.E, and Platen, E. (1991a). Relations between multiple ito and stratonovich integrals. *Stochastic Analysis and Applications*, **9**(3), 311–321.

Kloeden, P.E, and Platen, E. (1991b). Stratonovich and ito stochastic taylor expansions. *Mathematische Nachrichten*, **151**, 33–50.

Kloeden, P.E, and Platen, E. (1995). *Numerical Solution of Stochastic Differential Equations*. Springer-Verlag, New York.

Oksendal, B. (2000). *Stochastic Differential Equations: An Introduction with Applications*. 5th edn. Springer-Verlag, Berlin.

Platen, E. (1980). Weak convergence of approximations of ito integral equations. *Z Angew Math Mech*. **60**, 609–614.

Platen, E. and Bruti-Liberati, N. (2010). *Numerical Solution of Stochastic Differential Equations with Jumps in Finance.* Springer-Verlag, New York

Saito, Y, and Mitsui, T. (1993). Simulation of Stochastic Differential Equations. *The Annals of the Institute of Statistical Mathematics*, **3**, 419–432.

See Also

snssde1d and snssde2d for 1- and 2-dim sde.

sde.sim in package "sde". simulate in package "yuima".

Examples

```
## Example 1: Ito sde
## dX(t) = (2*(Y(t)>0)-2*(Z(t)<=0)) dt + 0.2 * dW1(t)
## dY(t) = -2*Y(t) dt + 0.2 * dW2(t)
## dZ(t) = -2*Z(t) dt + 0.2 * dW3(t)
## W1(t), W2(t) and W3(t) three independent Brownian motion
set.seed(1234)

fx <- expression(2*(y>0)-2*(z<=0) , -2*y, -2*z)
gx <- rep(expression(0.2),3)

mod3d1 <- snssde3d(x0=c(0,2,-2),drift=fx,diffusion=gx,M=500,Dt=0.003)
mod3d1
summary(mod3d1)
##
dev.new()
plot(mod3d1,type="n")
mx <- apply(mod3d1$X,1,mean)
my <- apply(mod3d1$Y,1,mean)
mz <- apply(mod3d1$Z,1,mean)
lines(time(mod3d1),mx,col=1)
lines(time(mod3d1),my,col=2)
lines(time(mod3d1),mz,col=3)
```

```
legend("topright",c(expression(E(X[t])),expression(E(Y[t])),
 expression(E(Z[t]))),lty=1,inset = .01,col=c(1,2,3),cex=0.95)
##
dev.new()
plot3D(mod3d1,display="persp") ## in space (O,X,Y,Z)

## Example 2: Stratonovich sde
## dX(t) = Y(t)* dt  + 0.2 o dW3(t)
## dY(t) = (4*( 1-X(t)^2 )* Y(t) - X(t))* dt + 0.2 o dW2(t)
## dZ(t) = (4*( 1-X(t)^2 )* Z(t) - X(t))* dt + 0.2 o dW3(t)
## W1(t), W2(t) and W3(t) are three correlated Brownian motions with Sigma

fx <- expression( y , (4*( 1-x^2 )* y - x), (4*( 1-x^2 )* z - x))
gx <- expression( 0.2 , 0.2, 0.2)
# correlation matrix
Sigma <-matrix(c(1,0.3,0.5,0.3,1,0.2,0.5,0.2,1),nrow=3,ncol=3)

mod3d2 <- snssde3d(drift=fx,diffusion=gx,N=10000,T=100,type="str",corr=Sigma)
mod3d2
##
dev.new()
plot(mod3d2,pos=2)
##
dev.new()
plot(mod3d2,union = FALSE)
##
dev.new()
plot3D(mod3d2,display="persp") ## in space (O,X,Y,Z)
```

`st.int` *Stochastic Integrals*

Description

The (S3) generic function `st.int` of simulation of stochastic integrals of Itô or Stratonovich type.

Usage

```
st.int(expr, ...)
## Default S3 method:
st.int(expr, lower = 0, upper = 1, M = 1, subdivisions = 1000L,
                type = c("ito", "str"), ...)

## S3 method for class 'st.int'
summary(object, at ,digits=NULL, ...)
## S3 method for class 'st.int'
time(x, ...)
## S3 method for class 'st.int'
mean(x, at, ...)
```

```
## S3 method for class 'st.int'
Median(x, at, ...)
## S3 method for class 'st.int'
Mode(x, at, ...)
## S3 method for class 'st.int'
quantile(x, at, ...)
## S3 method for class 'st.int'
kurtosis(x, at, ...)
## S3 method for class 'st.int'
min(x, at, ...)
## S3 method for class 'st.int'
max(x, at, ...)
## S3 method for class 'st.int'
skewness(x, at, ...)
## S3 method for class 'st.int'
moment(x, at,  ...)
## S3 method for class 'st.int'
cv(x, at,  ...)
## S3 method for class 'st.int'
bconfint(x, at,  ...)

## S3 method for class 'st.int'
plot(x, ...)
## S3 method for class 'st.int'
lines(x, ...)
## S3 method for class 'st.int'
points(x, ...)
```

Arguments

`expr`	an `expression` of two variables `t` (time) and `w` (`w`: standard Brownian motion).
`lower`, `upper`	the lower and upper end points of the interval to be integrate.
`M`	number of trajectories (Monte-Carlo).
`subdivisions`	the maximum number of subintervals.
`type`	Itô or Stratonovich integration.
`x`, `object`	an object inheriting from class `"st.int"`.
`at`	time between `lower` and `upper`. Monte-Carlo statistics of stochastic integral at time `at`. The default `at = upper`.
`digits`	integer, used for number formatting.
`...`	potentially further arguments for (non-default) methods.

Details

The function `st.int` returns a `ts` x of length `N+1`; i.e. simulation of stochastic integrals of Itô or Stratonovich type.

The Itô interpretation is:

$$\int_{t_0}^{t} f(s)dW_s = \lim_{N\to\infty}\sum_{i=1}^{N} f(t_{i-1})(W_{t_i} - W_{t_{i-1}})$$

The Stratonovich interpretation is:

$$\int_{t_0}^{t} f(s)\circ dW_s = \lim_{N\to\infty}\sum_{i=1}^{N} f\left(\frac{t_i + t_{i-1}}{2}\right)(W_{t_i} - W_{t_{i-1}})$$

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

Value

`st.int` returns an object inheriting from `class` `"st.int"`.

`X`	the final simulation of the integral, an invisible `ts` object.
`fun`	function to be integrated.
`type`	type of stochastic integral.
`subdivisions`	the number of subintervals produced in the subdivision process.

Author(s)

A.C. Guidoum, K. Boukhetala.

References

Ito, K. (1944). Stochastic integral. *Proc. Jap. Acad, Tokyo*, **20**, 19–529.

Stratonovich RL (1966). New Representation for Stochastic Integrals and Equations. *SIAM Journal on Control*, **4(2)**, 362–371.

Kloeden, P.E, and Platen, E. (1995). *Numerical Solution of Stochastic Differential Equations*. Springer-Verlag, New York.

Oksendal, B. (2000). *Stochastic Differential Equations: An Introduction with Applications*. 5th edn. Springer-Verlag, Berlin.

See Also

`snssde1d`, `snssde2d` and `snssde3d` for 1,2 and 3-dim sde.

Examples

```
## Example 1: Ito integral
## f(t,w(t)) = int(exp(w(t) - 0.5*t) * dw(s)) with t in [0,1]
set.seed(1234)

f <- expression( exp(w-0.5*t) )
mod1 <- st.int(expr=f,type="ito",M=50,lower=0,upper=1)
mod1
```

```
summary(mod1)
## Display
plot(mod1)
lines(time(mod1),apply(mod1$X,1,mean),col=2,lwd=2)
lines(time(mod1),apply(mod1$X,1,bconfint,level=0.95)[1,],col=4,lwd=2)
lines(time(mod1),apply(mod1$X,1,bconfint,level=0.95)[2,],col=4,lwd=2)
legend("topleft",c("mean path",paste("bound of", 95," percent confidence")),
       inset = .01,col=c(2,4),lwd=2,cex=0.8)

## Example 2: Stratonovich integral
## f(t,w(t)) = int(w(s)  o dw(s)) with t in [0,1]
set.seed(1234)

g <- expression( w )
mod2 <- st.int(expr=g,type="str",M=50,lower=0,upper=1)
mod2
summary(mod2)
## Display
plot(mod2)
lines(time(mod2),apply(mod2$X,1,mean),col=2,lwd=2)
lines(time(mod2),apply(mod2$X,1,bconfint,level=0.95)[1,],col=4,lwd=2)
lines(time(mod2),apply(mod2$X,1,bconfint,level=0.95)[2,],col=4,lwd=2)
legend("topleft",c("mean path",paste("bound of", 95," percent confidence")),
       inset = .01,col=c(2,4),lwd=2,cex=0.8)
```

TEX.sde *Converting Sim.DiffProc Objects to LaTeX*

Description

These methods produces the related LaTeX table and mathematic expression for Sim.DiffProc environment.

Usage

```
TEX.sde(object, ...)

## Default S3 method:
TEX.sde(object, ...)
```

Arguments

`object`	an objects from class `MCM.sde` and `MEM.sde`. Or an R vector of `expression` of SDEs, i.e., drift and diffusion coefficients.
`...`	arguments to be passed to `kable` function if `object` from class `MCM.sde`.

Details

New tools for constructing tables and mathematical expressions with `Sim.DiffProc` package.

An overview of this package, see `browseVignettes('Sim.DiffProc')` for more informations.

Author(s)

A.C. Guidoum

References

Guidoum AC, Boukhetala K (2020). "Performing Parallel Monte Carlo and Moment Equations Methods for Itô and Stratonovich Stochastic Differential Systems: R Package Sim.DiffProc". *Journal of Statistical Software*, **96**(2), 1–82. doi:10.18637/jss.v096.i02

Xie Y (2015). Dynamic Documents with R and knitr. 2nd edition. Chapman and Hall/CRC, Boca Raton, Florida. ISBN 978-1498716963, URL `https://yihui.org/knitr/`.

Wickham H (2015). Advanced R. Chapman & Hall/CRC The R Series. CRC Press. ISBN 9781498759809.

See Also

`kable` create tables in LaTeX, HTML, Markdown and reStructuredText.

`toLatex` converting R Objects to BibTeX or LaTeX.

Examples

```
## LaTeX mathematic for an R expression of SDEs
## Copy and paste the following output in your LaTeX file

# Example 1

f <- expression(-mu1 * x)
g <- expression(mu2 * sqrt(x))
TEX.sde(object = c(drift = f, diffusion = g))

# Example 2

f <- expression(mu1*cos(mu2+z),mu1*sin(mu2+z),0)
g <- expression(sigma,sigma,alpha)
TEX.sde(object = c(drift = f, diffusion = g))

## LaTeX mathematic for object of class 'MEM.sde'
## Copy and paste the following output in your LaTeX file

# Example 3

mem.mod3d <- MEM.sde(drift = f, diffusion = g)
TEX.sde(object = mem.mod3d)

## LaTeX table for object of class 'MCM.sde'
## Copy and paste the following output in your LaTeX file
```

```
# Example 4

## Not run:
mu1=0.25; mu2=3; sigma=0.05; alpha=0.03
mod3d <- snssde3d(drift=f,diffusion=g,x0=c(x=0,y=0,z=0),M=100,T=10)

stat.fun3d <- function(data, i){
   d <- data[i,]
   return(c(mean(d$x),mean(d$y),mean(d$z),
            var(d$x),var(d$y),var(d$z)))
}
mcm.mod3d = MCM.sde(mod3d,statistic=stat.fun3d,R=10,parallel="snow",ncpus=parallel::detectCores(),
                    names=c("m1","m2","m3","S1","S2","S3"))

TEX.sde(object = mcm.mod3d, booktabs = TRUE, align = "r", caption ="\LaTeX~
         table for Monte Carlo results generated by \code{TEX.sde()} method.")

## End(Not run)
```

Index